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Exact dynamic/static stiffness matrices of non-symmetric thin-walled beams considering coupled shear deformation effects

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Abstract

A general theory is proposed for the shear deformable thin-walled beam with non-symmetric open/closed cross-sections and its exact dynamic and static element stiffness matrices are evaluated. For this purpose, an improved shear deformable beam theory is developed by introducing Vlasov's assumption and applying Hellinger–Reissner principle. This includes the shear deformations due to the shear forces and the restrained warping torsion and due to the coupled effects between them, rotary inertia effects and the flexural–torsional coupling effects due to the non-symmetric cross-sections. Governing equations and force–deformation relations are derived from the energy principle and a system of linear eigenproblem with non-symmetric matrices is constructed based on 14 displacement parameters. And then explicit expressions for displacement parameters are derived and the exact dynamic and the static stiffness matrices are determined using force–deformation relationships. In order to verify the validity and the accuracy of this study, the numerical solutions are presented and compared with other numerical solutions available in the literature and results using the thin-walled beam element and the shell element. Particularly the influences of the coupled shear deformation on the vibrational and the elastic behavior of non-symmetric beams with various boundary conditions are investigated.

Keywords: Thin-walled beam; Coupled shear deformation; Exact dynamic and static stiffness

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1. Introduction

There is a growing interest in the development of reliable beam models, which can be used to analyze thin-walled structures. Thin-walled beams have been widely used in many civil, mechanical and aerospace engineering applications, both in their stand-alone forms and as stiffeners for plate and shell structures. Investigation into the vibrational and the elastic behavior of thin-walled beams with open and closed cross-sections has been carried out extensively since the early works of Vlasov [1] and Timoshenko and Gere [2]. Particularly monographs by Kollbrunner and Basler [3] and Gjelsvik [4] are worth remarking as useful references for the thin-walled beam theory and its applications. Also intensive research works [5–10] have been made over the years to develop finite element models that can accurately represent the complex flexural–torsional warping deformable response of these structures.

Early attempts to present a FE formulation about a flexural behavior of thin-walled beams were based on the classical Euler–Bernoulli beam theory, resulting in the so called C^1-type beam elements. Thereafter, with the deepening of the research work, great attention was paid to the shear flexible modeling approaches, so called C^0-type beam elements [5,6,8,9].

In torsional problem of thin-walled beam, the previous researches [11–15] were based on the theory of Vlasov [1] considering the restrained warping effect, but have neglected the warping shear deformation. For instance, the torsional stiffness matrix was derived using the homogeneous solution for the governing differential equations [13].

Since then a general finite element formulation including the shear deformations due to the shear forces and the restrained warping was developed by utilizing hybrid model [16,17] and displacement-based model [18]. Gendy et al. [17] developed two straight beam elements with linear and quadratic displacement assumption, respectively, for the elastic analysis of thin-walled beam. That formulation is valid for both open and closed cross-sections and this is accomplished by using a kinematical description accounting for both flexural and warping torsional effects. However, these researchers included only the shear deformations due to the shear forces and restrained warping, but did not take into account the coupled shear deformation (CSD) between them.

It is well known that the coupling between the flexure and the torsion is produced in case of thin-walled beam with non-symmetric cross-section. Also the shear deformation effects between the shear forces and the restrained warping are coupled. In the development of CSD between the flexural and the torsional problems and the finite element model of the thin-walled beams for the dynamic and the static analyses of the ship hull, a significant contribution was made by a few researchers [19–22]. Gunnlaugsson and Pedersen [19] derived the governing differential equations for coupled responses of thin-walled beams and established the stiffness matrix of the prismatic element, but the homogeneous solutions to the governing equations for the special uncoupled case of a doubly symmetric cross-section was used as interpolation functions. Ref. [20] revealed that the influence of the higher order warping modes of the cross-section on the overall response analysis of the thin-walled beam was not significant. Hu et al. [21] derived the fourth-order governing differential equations for thin-walled beams with asymmetric cross-sections under coupled bending and torsion