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Theory of initially twisted, composite, thin-walled beams

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Abstract

An asymptotically correct theory for initially twisted, thin-walled, composite beams has been constructed by the variational asymptotic method. The strain energy of the original, three-dimensional structure is first rigorously reduced to be a two-dimensional energy expressed in terms of shell strains. Then the two-dimensional strain energy is further reduced to be expressed in terms of the classical beam strain measures. The resulting theory is a classical beam model approximating the three-dimensional energy through the first-order of the initial twist. Consistent use of small parameters that are intrinsic to the problem allows a natural derivation for all thin-walled beams within a common framework, regardless of whether the section is open, closed, or strip-like. Several examples are studied using the present theory and the results are compared with a general cross-sectional analysis, VABS, and other published results.

Keywords: Composite beams; Thin-walled; Initial twist; Asymptotic method; VABS

1. Introduction

A thin-walled beam is characterized as a flexible body that has different magnitudes for all three of its characteristic dimensions [1]. To be classified as a beam, c, the characteristic dimension of the cross-section, must be much smaller than l, the wavelength

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of the deformation along the beam, i.e. $c/l \ll 1$. Moreover, for a beam to be classified as thin-walled implies that the maximum thickness of the walls, h, is much smaller than c, so that $h/c \ll 1$. Although one can analyze thin-walled beams using three-dimensional (3D) elasticity theory, thin-walled beam theories take advantage of the small parameters, h/c and c/l, to derive a one-dimensional (1D) model. This model consists of 1D constitutive equations (cross-sectional elastic constants) and 'recovery relations'. The former are used in the 1D equilibrium and kinematic equations to analyze the original 3D structure, and the latter provide approximate values of the 3D displacement, strain, and stress from the 1D solution.

Thin-walled beam theories strive to present closed-form expressions for cross-sectional stiffness constants and stresses (or stress flows). There are mainly two types of thin-walled beam theories. The first type can be classified as ad hoc models [1–5]. In these models, assumptions are invoked based on engineering intuition. These can be assumptions that the beam deforms in specific modes or that certain components of the displacement/strain/stress are negligible. Usually, these assumptions are based on experience with thin-walled beams made with isotropic materials, which can be justified by some exact solutions. However, for anisotropic materials, various modes of deformation can be coupled, and these theories might fail for some special cases which cannot be properly represented by the invoked assumptions [6]. Nevertheless, some of these models such as [4,5] can provide a good prediction for many cases, and it is straightforward to refine the model by incorporate additional deformation such as transverse shear to remedy possible errors introduced by ad hoc assumptions.

The second type encompasses asymptotic models [6–9]. Therein, the original 3D elasticity equations are mathematically reduced to a 1D model using small parameters inherent to the problem. While application of traditional asymptotic methods is possible, the authors prefer the Variational Asymptotic Method (VAM) [10]. In these models, the material anisotropy is accounted for in a consistent and systematic manner, and those deformation modes that contribute most significantly to the energy emerge naturally. In our formulation, elastic couplings among all deformations are accounted for by using the 3D material law, which uses 21 elastic constants for anisotropic materials. However, the refined models constructed directly using the VAM are of little practical use, perse. Usually, some transformation, which might detract from the asymptotical correctness, has to be carried out to convert such models into a form that is of practical use for engineers [11].

The present paper was originally planned to serve as a natural extension of the work in [9] to enhance the capability of that theory to accommodate initial twist, so that more realistic problems (such as pretwisted composite rotor blades or wind turbines) can be analyzed. It was later found out that it is very complicated, if not impossible, to incorporate the initial twist into that, already complex, formulation. Instead, the present formulation is cast in an intrinsic form and the derivation departs from previous work at the outset. First, the 3D elasticity representation is rationally reduced to the classical shell approximation of Berdichevsky [10] with geometric correction by considering h/c as the main small parameter and taking into account all first-order corrections from the initial twist of the thin-walled beam. Then the two-dimensional (2D) variables are expressed in terms of intrinsic beam variables and unknown warping functions. Substituting these relations back