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A generalized Vlasov theory for composite beams

Wenbin Yu[a,*], Dewey H. Hodges[b], Vitali V. Volovoi[b], Eduardo D. Fuchs[b]

[a]*Department of Mechanical and Aerospace Engineering, Utah State University, Logan, UT 84322-4130, USA*
[b]*School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150, USA*

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Abstract

A generalized Vlasov theory for composite beams with arbitrary geometric and material sectional properties is developed based on the variational asymptotic beam sectional analysis. Instead of invoking *ad hoc* kinematic assumptions, the variational-asymptotic method is used to rigorously split the geometrically-nonlinear, three-dimensional elasticity problem into a linear, two-dimensional, cross-sectional analysis and a nonlinear, one-dimensional, beam analysis. The developed theory is implemented into VABS, a general-purpose, finite-element based beam cross-sectional analysis code. Several problems are studied to compare the present theory with published results and a commercial three-dimensional finite element code. The present work focuses on the issues concerning the use of the Vlasov correction in the context of the accuracy of the resulting beam theory. The systematic comparison with three-dimensional finite element analysis results helps to quantitatively demonstrate both the advantages and limitations of the Vlasov theory.

Keywords: Vlasov; Composite beams; Asymptotic method; Thin-walled; VABS

1. Introduction

For thin-walled beams with open sections it is well known that the classical beam theory, which relies on four generalized strain measures associated with stretching of the reference line (γ_{11}), twist (κ_1), and bending in two mutually orthogonal directions (κ_2 and κ_3), does not suffice; and a refined beam theory becomes necessary. There are several ways

* Corresponding author. Tel.: +1 435 797 8246; fax: +1 435 797 2417.
E-mail address: wenbin.yu@usu.edu (W. Yu).

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to explain such a phenomenon. Perhaps the most revealing one is from the standpoint of the St. Venant principle. One implication of the principle for beams is that any two statically equivalent systems of forces at the end of a long beam provide practically identical stress distributions far away from that end. More precisely, the difference between the two stress distributions exponentially decays along the axis of the beam. This provides validity for the classical beam theory since its generalized strain measures adequately describe the non-decaying part of the three-dimensional (3-D) elasticity solution. As a result, for static and low-frequency behavior of slender beams that are not thin-walled with open section (what we will call 'regular beams'), classical theory is adequate. Technically speaking, the principle remains valid even for *t*hin-*w*alled, *o*pen-*s*ection (TWOS) beams. However, application of a certain system of forces (usually referred to as the 'bi-moment') at an end of a TWOS beam leads to a deformation mode that decays away from that end much more slowly than any system of forces in a regular beam. This implies that in most practical applications, even for relatively long TWOS beams with slenderness ratios of 50 or more, the importance of this additional decaying mode may remain significant [1].

Engineering theories that adequately model this effect for isotropic beams existed for a good part of the last century. They rely on incorporation of the derivative of twist (κ_1') as an independent generalized strain measure. Most commonly such a refinement is referred to as Vlasov theory [2,3]; however, alternative names, such as Wagner theory [4], are in use as well. Because the resulting governing equation for torsion in Vlasov theory is of fourth order, rather than second order as in the St. Venant treatment of torsion, an additional boundary condition is required at each end of the beam. The geometric form of this boundary condition, *i.e.* specifying $\kappa_1=0$ at a boundary, is often referred to as 'restrained warping.' Indeed, within the context of this theory, only warping out of the cross-sectional plane is present and its magnitude is proportional to κ_1. This leads to other common names of such a refinement: 'torsional theory with restrained warping' (as opposed to the St. Venant torsional theory where the warping is free) and 'nonuniform torsion' (as opposed to uniform torsion in the St. Venant case). While those theories were based primarily on engineering intuition, it was later rigorously shown that the slowly decaying deformation mode in question is indeed related to torsion [5,6].

Primarily because the rotorcraft industry uses composite, TWOS beam structures in such parts as bearingless rotor flexbeams, extension of the Vlasov theory to anisotropic beams has attracted significant attention from researchers [7–10]. Such theories construct beam models based on the classical, laminated plate/shell theory in conjunction with the kinematic assumptions that were originally used in Vlasov theory for isotropic beams. In particular, the beam cross-section is assumed to be rigid in its own plane, and the transverse shear strains are neglected [7,8]. As discussed in detail in [6], such assumptions lead to certain contradictions even for isotropic beams, while the consequences are even less predictable for the generally anisotropic case. There exists an alternative approach to constructing thin-walled beam theories that avoids *ad hoc* kinematic assumptions and relies instead on equilibrium equations. This in general leads to more rigorous thin-walled beam theories [11,12]. Some attempts to apply this method to the development of Vlasov theory have been made [10], but the procedure is not straightforward because there are not enough equilibrium equations to solve for all the necessary quantities.