ARTICLE IN PRESS

Journal of Mathematical Psychology ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at SciVerse ScienceDirect

Journal of Mathematical Psychology

journal homepage: www.elsevier.com/locate/jmp

The influence of power law distributions on long-range trial dependency of response times

Espen A.F. Ihlen

Department of Neuroscience, Norwegian University of Science and Technology, N-7489 Trondheim, Norway

HIGHLIGHTS

- A definition of long-range dependency for power law distributed RT is introduced.
- Method to identify α-stable and long-range trial dependent RT is developed.
- Word naming RT is α-stable distributed and time independent.
- Simple RT is α-stable distributed and time independent.
- Power law and long-range dependent RTs are not assessed by normal scaling analyses.

ARTICLE INFO

Article history:
Received 7 May 2012
Received in revised form
1 July 2013
Available online xxxx

Keywords:
Reaction times
$1/f$ noise
Power law
Multifractal
Variability
Long-range dependency

ABSTRACT

Empirical response time distributions from simple cognitive tasks are typically unimodal and positively skewed. In contrast, variance based scaling analyses, which have been used to study long-range dependency via the Hurst exponent, $H > 0.5$, assume Gaussian response time distributions. This article presents a general method which can identify long-range trial dependency for response time series with power law distributions. The method fits an α-stable distribution to the response time series which satisfies a general version of the central limit theorem and consequently, an α-stable extension ($H_{q=0} > 1/\alpha$) of long-range dependency. The method was used to reanalyze 96 response time series from three existing data sets which included simple reaction time, word naming, choice decision, and interval estimation tasks. The results showed that all response time distributions were appropriately modelled by an α-stable distribution. Furthermore, the response time series from the simple response and word naming tasks were not long-range dependent when the α-stable definition $H_{q=0} > 1/\alpha$ was used in place of the Gaussian response time distribution definition $H_{q=2} > 0.5$. The deviation between the two definitions of long-range dependency was shown to be caused by divergence of the variance for response time distributions with power-law decaying tails. The study concludes that the new α-stable definition, $H_{q=0} > 1/\alpha$, of the long-range trial dependency should be used in the research of response time series instead of the Gaussian definition, $H_{q=2} > 0.5$.

1. Introduction

The analyses of response time series have provided insight into the mental organization and cognitive processes used in a wide variety of tasks such as simple reaction time, word naming, choice decision, visual search, memory search and lexical decision. Historically, the mean and median values have been used to draw statistical inferences from response time series, but in the past few decades these parameters have been superseded by models of the entire distribution of the response time series (Luce, 1986). Response time distributions are typically unimodal and positively skewed towards long response latencies and consequently, violate the symmetry assumption of the Gaussian distribution. Therefore, non-Gaussian distributions like the ex-Gaussian (Burbeck & Luce, 1982), shifted Wald (Schwarz, 2001), log-normal (Ulrich & Miller, 1993), gamma (Van Zandt & Ratcliff, 1995), Weibull (Logan, 1992) and power law distributions (Moscoso del Prado Martin, 2009) have been suggested as better models of response time distributions. In contrast to the suggested models above, this article introduces an α-stable distribution which can incorporate both power law decaying tails of response time distributions and long-range trial dependency (LRD) of response times.

The ex-Gaussian and shifted Wald distribution have been suggested as simple and convenient models of response time distributions. Several two-component theories of cognitive performance have been proposed to motivate an ex-Gaussian distribution (e.g., Balota & Spieler, 1999; Penner-Wilger, Leth-Steensen, & LeFevre, 2002; Rohrer & Wixted, 1994), but the a priori theoretical

E-mail address: espen.ihlen@ntnu.no.

0022-2496/$ – see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.jmp.2013.07.001

argument for the choice of the ex-Gaussian distribution has been debated (Matzke & Wagenmakers, 2009). In contrast, the shifted Wald distribution has a more robust a priori theoretical argument compared to the ex-Gaussian distribution due to the relation to the diffusion model for response time in the choice decision task (Ratcliff, 1978; Schwarz, 2001). The shifted Wald distribution is the first passage time distribution of a diffusion process towards an absorbing boundary where its parameters define the distance to the absorbing boundary (i.e., response caution), the linear drift of the diffusion process (i.e., task difficulty) and the non-decisional response time.

The ex-Gaussian and the shifted Wald models share the common assumption of an exponentially decaying right tail of the response time distribution. Some studies have suggested response time distributions with slower than exponential decays at the tails, also referred to as heavy tailed and power law distributions (Holden, Van Orden, & Turvey, 2009; Moscoso del Prado Martin, 2009). The Weibull or stretched exponential distribution defines a continuum between the exponential and the Gaussian distribution and has been suggested as a model of the competitive contest between memory and the initiation of task performance in memory search response times (Logan, 1992) and visual search response times (Palmer, Horowitz, Torralba, & Wolfe, 2011). However, a power law distribution called the Pareto distribution was found to provide a better fit to response time distributions compared to both the heavy tailed Weibull and the log-normal distribution to a large sample of RTs collected via the web (Moscoso del Prado Martin, 2009). Furthermore, a recent study introduced a composite model composed of a log-normal distribution and a power law distribution as an appropriate model for response time distributions in word naming tasks (Holden et al., 2009). The same study associated heavy tailed distributions with multiplicative interactions between the components of mental organization motivated by the physical theories of self-organization in complex systems (Van Orden, Holden, & Turvey, 2003). However, none of the heavy tailed distributions above provides a way to define LRD. The latter is another concept in the physics of self-organization in a complex system. This article introduces an α-stable distribution that defines LRD for power law distributed response times series.

The statistical inference of response time series by a model of its distribution assumes that the individual response times are independent random variables. Several studies of response time series from cognitive tasks such as the simple reaction time, word naming, choice decision, visual search, memory search, and lexical decision have reported LRD of response times over hundreds of trials (Gilden, 2001; Van Orden et al., 2003). The LRD of response times can be generated by a superposition or interaction of subprocesses which evolve on multiple time scales (Van Orden et al., 2003; Wagenmakers, Farrell, & Ratcliff, 2004). These multiple time scales of response time series have been associated with theoretical constructs such as level of consciousness (Ward, 2002) and mental sets (Gilden, 2001). Although there is controversy as to whether response times are long-range or short-range trial dependent, there is consensus that response time series possess some form of trial dependency (Farrell, Wagenmakers, & Ratcliff, 2006; Van Orden, Holden, & Turvey, 2005; Wagenmakers, Farrell, & Ratcliff, 2005). LRD and other forms of trial dependency cannot be identified by conventional parameters such as mean and variance or by a model of the response time distribution. Therefore, scaling analyses of the response time series in time and frequency domains has gained interest for inference and modelling of response time dynamics.

The LRD of response times is numerically defined by a scaling exponent, the Hurst exponent, denoted H, obtained by analyses such as detrended fluctuation analysis, scaled window variance analysis or rescaled range analysis, to mention but a few (cf. Delignières et al., 2006). All of these analyses numerically define LRD as a dimensionless exponent H by scaling the variance, standard deviation or root-mean-square of the response time series over s number of trials. However, the numerical definition of the scaling exponent H assumes that the response time distributions are Gaussian like and that the second order moment (i.e., $E(x^2)$) of the response times are well defined. Power law decaying tails of the response time distribution lead to divergence in the second moment of response times (i.e., $E(x^2) \rightarrow \infty$ when the number of trials $N \rightarrow \infty$) and consequently, the scaling exponent H also diverges. Thus, stochastic models of trial-dependent dynamics in response time series, such as the aggregated sum of autoregressive processes (Ward, 2002) and fractional Gaussian noise (Gilden, 2001), assume that response time distributions have faster than exponential decaying tails similar to a Gaussian distribution. To overcome this fundamental problem of conventional variance-based scaling analyses, a generalized scaling analysis should be employed that considers other q-order moments when the response time distribution is positively skewed with power law decaying tails.

The aim of the present study was to obtain a general power law definition of LRD of response time series by modelling the response time distribution as an α-stable distribution and estimating the scaling exponents from other q-order moments. The organization of the article is as follows. First, I define the fundamental concepts of power laws and scale invariance prior to introducing the LRD concept. Second, I introduce the class of α-stable processes and describe its use in constructing non-Gaussian definitions of LRD based on extensions of variance-based scaling analyses. Third, I describe a four-step method to identify LRD for response time series that are appropriately modelled by α-stable distributions. Finally, I applied the four-step method to reanalyze three sets of response time series data which were reported to be long-range dependent under the assumption of Gaussian distributed response times. To conclude, I discuss the possible origins of α-stable response time distributions and LRD.

2. Methods

2.1. Scale-invariance and power laws

The definition of scale-invariance states that the exponent α of a power law function, $f(x) = Ax^{\alpha}$, is invariant to coordinate transformation of the form $x = sy$ (cf. Gisiger, 2001). Consequently, the shape of the coordinate transformed power law function, $f(sy) = (As^{\alpha})y^{\alpha}$, remains the same by "zooming in" (i.e., $s < 1$) and "zooming out" (i.e., $s > 1$) on $f(sy)$. The constant, s, is referred to as the scale of the power law function, $f(sy)$. The psychological literature on scale invariance refers to two different types of power law functions for a response time series. The power law function $f(x)$ is (1) the probability distribution of response times x, or (2) the spectral density of the response time series where x is the spectral frequency or the second moment of the response time x. Power laws of type (1) are models of the response time distribution whereas power laws of type (2) define a dynamical property called LRD. In the response time literature to date, the definition of type (2) power laws disregard the fact that the conventional definition of LRD assumes a Gaussian distribution which are not scale invariant.

The next subsections introduce a method that joins the two concepts of scale invariance in type (1) and (2) power laws.

2.2. Long-range dependency (LRD)

LRD is related to scale invariance of the response time series itself. Scale invariance of the response time series x_k with trial $k = 1, 2, \ldots, n$ holds when the random walk profile $X_i = \sum_{k=1}^{i}(x_k - \bar{x})$