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Note

Performance assessment of airlines using range-adjusted measure, strong complementary slackness condition, and discriminant analysis

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ABSTRACT

This study integrates RAM (range-adjusted measure), SCSC (strong complementary slackness condition), and DEA–DA (data envelopment analysis–discriminant analysis) to rank airlines. As conventional DEA models do not fully use all inputs and outputs, they result zero in many multipliers. These sorts of DEA models may yield many efficient decision-making units (DMUs). This decreases the discrimination power of DEA. To overcome this limitation, this study proposes a novel application of RAM–DEA/SCSC along with DA. A case study demonstrates the applicability of our proposed approach.

1. Introduction

This paper proposes a novel data envelopment analysis (DEA) model for evaluating the performance of Iran's airlines. Our proposed model not only ranks all Iranian airlines but also fully uses all inputs and outputs. To prevent alternative solutions, Sueyoshi and Sekitani (2007) integrated DEA and strong complementary slackness condition (SCSC) and proposed the DEA/SCSC model. Nonetheless, the DEA/SCSC model does not guarantee that the ties among efficient decision-making units (DMUs) are broken. To rank efficient DMUs, Sueyoshi and Sekitani (2007) used DEA–discriminant analysis (DEA–DA). Barros and Wanke (2015) used the technique for order of preference by similarity to ideal solution (TOPSIS) to assess the relative efficiency of African airlines. Merkert and Pearson (2015) developed a new approach for measuring the impact of an airline's customer service on profit. To calculate the efficiency scores of airlines (DMUs), this paper integrates one of the DEA models called range-adjusted measure (RAM) and SCSC. To rank DMUs, DEA–DA is applied.

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2. Methodology

2.1. Steps of calculations

In this paper, first, the RAM model and the SCSC concept are combined. The main objective of our proposed RAM-DEA/SCSC model is to classify all DMUs into efficient and inefficient groups so that all multipliers of efficient DMUs become positive. Then, the two groups of DMUs are separated using the DEA–DA model to minimize misclassification. As a result, unique optimal solutions are calculated by adjusted efficiency score. To reduce the number of efficient DMUs, we combine RAM/SCSC and DEA–DA. Thus, our proposed approach can identify the best DMU.

2.2. Primal and dual of RAM-DEA model

Here, we review the RAM model and propose a new version of the RAM model. Suppose we have n DMUs (DMU_j; $j = 1, 2, \ldots, n$). Let $x_j = (x_{1j}, x_{2j}, \ldots, x_{ij})^T > 0$ and $y_j = (y_{1j}, y_{2j}, \ldots, y_{rj})^T > 0$ denote the input and output vectors of the jth DMU, respectively. Two vectors (d_i^x and d_r^y) represent the input and output slacks, respectively. The superscript T stands for a vector transpose. The subscripts i and r show the ith input ($i = 1, 2, \ldots, m$) and the rth output ($r = 1, 2, \ldots, s$), respectively. The subscript k shows the DMU under evaluation. The RAM model for assessing the relative efficiency of the kth DMU is as follows (Cooper et al., 1999):

$$\begin{aligned} & maz\ Z=\sum_{i=1}^{m}R_i^x d_i^x+\sum_{r=1}^{s}R_r^y d_r^y \\ & \sum_{j=1}^{n}x_{ij}\lambda_j+d_i^x=x_{ik}\quad i=1,\dots,m \\ & \sum_{j=1}^{n}y_{rj}\lambda_j-d_r^x=y_{rk}\quad r=1,\dots,s \\ & \sum_{j=1}^{n}\lambda_j=1 \\ & d_i^x,d_r^x,\lambda_j\geq 0\quad j=1,\dots,n \end{aligned} \tag{1}$$

where $\lambda=(\lambda_1,\ \dots,\ \lambda_n)^T$ refers to the "intensity" variable. They are used to link the input and output vectors by a convex combination. The ranges in Model (1) are calculated by upper and lower bounds on inputs and outputs. These upper and lower bounds are presented as follows:

$$\begin{aligned} R_i^x &= \frac{1}{(m+s)(max\{x_{ij}|j=1,\dots,n\}-\min\{x_{ij}|j=1,\dots,n\})} \\ R_r^y &= \frac{1}{(m+s)\left(max\left\{y_{rj}\middle|j=1,\dots,n\right\}-\min\left\{y_{rj}\middle|j=1,\dots,n\right\}\right)} \end{aligned} \tag{2}$$

The optimal efficiency score of DMU under evaluation can be determined as follows:

$$\theta^*=1-\left[\sum_{i=1}^{m}R_i^x d_i^x+\sum_{r=1}^{s}R_r^y d_r^y\right] \tag{3}$$

where d_i^x and d_r^y are slack variables and represent the level of inefficiency. The optimal efficiency score is calculated by subtracting the level of inefficiency from unity. The dual formulation of Model (1) is as follows:

$$\begin{aligned} & \min P=\sum_{i=1}^{m}v_i x_{ik}-\sum_{r=1}^{s}u_r y_{rk}+\sigma \\ & s.t \\ & \sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}+\sigma\geq 0,\quad j=1,\dots,n \\ & v_i\geq R_i^x,\quad i=1,\dots,m \\ & u_r\geq R_r^y,\quad r=1,\dots,s \\ & \sigma,v_i,u_r:URS \end{aligned} \tag{4}$$

where v_i and u_r represent all dual variables related to the first and second set of constraints in Model (1). The dual variable σ is obtained from the third constraint of Model (1).

2.3. RAM–DEA/SCSC

Given the complementary slackness condition (CSC), correlations between the optimal solution of Model (1) (σ^*, λ^*, d_i^x, d_r^y) and the optimal solution of Model (4) (Z^*,v^*,u^*) are shown as follows (Bazaraa et al., 2010):

$$\lambda_j^*\left(\sigma^*+v^*x_j+u^*y_j\right)=0\quad(j=1,\dots,n) \tag{5}$$

$$d_i^{x*}v_i^*=0\quad(i=1,\dots,m) \tag{6}$$

$$d_r^{y*}u_r^*=0\quad(r=1,\dots,s) \tag{7}$$

Both the optimal solutions of Model (1) and optimal solutions of Model (4) are satisfied in the following conditions:

$$\lambda_j^*+\left(\sigma^*+v^*x_j+u^*y_j\right)>0\quad(j=1,\dots,n) \tag{8}$$

$$d_i^{x*}+v_i^*>0\quad(i=1,\dots,m) \tag{9}$$

$$d_r^{y*}+u_r^*>0\quad(r=1,\dots,s) \tag{10}$$

Here, we combine Model (1) and Model (4) as follows:

$$\begin{aligned} & \text{Max}\ \eta \\ & \text{s.t.} \\ & \sum_{j=1}^{n}x_{ij}\lambda_j+d_i^x=x_{ik}, \\ & \sum_{j=1}^{n}y_{rj}\lambda_j-d_r^y=y_{rk}, \\ & \sum_{j=1}^{n}\lambda_j=1 \\ & \sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}+\sigma\geq 0 \\ & \sum_{i=1}^{m}R_i^x d_{ik}^x-\sum_{r=1}^{s}R_r^y d_{rk}^y=\sum_{i=1}^{m}v_i x_{ik}-\sum_{r=1}^{s}u_r y_{rk}+\sigma,\quad j=1,\dots,n \\ & \lambda_j+\sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}+\sigma\geq\eta e^T,\quad j=1,\dots,n \\ & v_i\geq R_i^x,\quad i=1,\dots m \\ & u_r\geq R_r^y,\quad r=1,\dots s \\ & v_i+d_i^x\geq\eta e^T,\quad i=1,\dots m \\ & u_r+d_r^y\geq\eta e^T,\quad i=1,\dots s \\ & d_i^x,d_r^x,\lambda_j,x_i,y_r,\eta\geq 0;\quad\sigma,v_i,u_r:URS;\quad j=1,\dots,n \end{aligned} \tag{11}$$

The fifth constraint of Model (11) ensures that the objective function of Model (1) is equivalent to the objective function of Model (4). The last constraints of Model (11) are related to SCSC (5)–(10). The unit vector is represented by $e=(1,1,\ \dots\ ,1)$. A new decision variable (η) is added to Model (11) to keep SCSC optimal.

2.4. Review of characteristics of supporting hyperplane in DEA

Sueyoshi and Goto (2011) characterized the supporting hyperplane mathematically by the following proposition:

Proposition 1. A supporting hyperplane of DMU_k is as follows:

$$\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}+\sigma\geq 0,\quad j=1,\dots n \tag{12}$$

Here, v_i ($i=1,\ \dots,\ m$) and u_r ($r=1,\ \dots,\ s$) are parameters for indicating the direction of a supporting hyperplane, and σ indicates the intercept of the supporting hyperplane. The parameters are unknown and should be measured by the following equations:

$$\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}+\sigma=0,\quad j\in R_k \tag{13}$$

where R_k stands for a reference set of the kth DMU, for which operational performance is measured by Expression (13). Sueyoshi and Goto (2011) indicated that Proposition 1 characterizes a supporting hyperplane in data space. The proposition shows how the