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The impact of highly and minimally guided discovery instruction on promoting the learning of reasoning strategies for basic add-1 and doubles combinations

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ABSTRACT

A 9-month training experiment was conducted to evaluate the efficacy of highly and minimally guided discovery interventions targeting the add-1 rule (the sum of a number and one is the next number on the mental number list) and doubles relations (e.g., an everyday example of the double 5 + 5 is five fingers on the left hand and five fingers on the right hand make 10 fingers in all) and to compare their impact with regular classroom instruction on adding 1 and the doubles. After pretest, 81 kindergarten to second-grade participants were randomly assigned to one of three training conditions: highly guided add-1 training, highly guided doubles training, or minimally guided add-1 and doubles practice. The highly guided add-1 training served as an active control for the highly guided doubles training and vice versa, and the minimally guided practice condition served to control for the impact of extra practice. ANCOVAs using pretest score and age as covariates indicated that both highly guided and minimally guided interventions were successful in promoting retention and transfer for the relatively salient add-1 rule, but only highly guided training produced transfer for the less-salient doubles strategies. The findings indicate that the degree of guidance needed to achieve fluency with different addition reasoning strategies varies.

Q4 Introduction

The Common Core State Standards (CCSS; Council of Chief State School Officers [CCSSO], 2010) lay a framework for identifying the central skills and concepts pupils need to master at each grade level. CCSS Standard 6 in the grade 1 operations and algebraic thinking domain states: "Add and subtract within 20, demonstrating fluency for addition . . . within 10" and use reasoning strategies to determine sums. Fluency implies *efficient* (accurate and fast) production of sums. As used hereafter, the term also means *appropriate* and *adaptive* application of knowledge (e.g., selective application of a rule/strategy to novel problems not previously solved). Although there is general agreement that *all* children need to achieve fluency with basic sums (CCSSO, 2010; National Council of Teachers of Mathematics [NCTM], 2000, 2006; National Mathematics Advisory Panel [NMAP], 2008; National Research Council [NRC], 2001), there is disagreement about the best method(s) for achieving this goal. The main aim of this study was to gauge the efficacy of software designed to promote primary grade pupils' fluency with the most basic sums—the starting points of mental-addition fluency. A by-product of the research was comparing the relative efficacy of different instructional approaches as a step toward identifying best practices in mathematics education.

Instructional content of the interventions: why focus on reasoning strategies?

Reasoning strategies in general

The *meaningful learning* of a basic sum or family of basic sums entails three overlapping phases (Verschaffel, Greer, & De Corte, 2007). Initially, children use object or verbal counting to determine the sum (Phase 1: counting strategies). For example, for 2 + 3, a child would typically count, "Three, four is 1 more, *five* is two more—the answer is five." Then, as a result of discovering patterns or relations, children invent reasoning strategies, which they apply consciously and relatively slowly (Phase 2: deliberate reasoning strategies). For

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example, children may discover that adding 1 is related to their existing knowledge of number-after relations. The discovery of this connection leads to the invention of a strategy, namely the *add-1 rule*. This reasoning strategy or rule specifies that the sum of any whole number, *n*, and 1 (or 1 + *n*)—but not other items—is the number after *n* in the count sequence (e.g., the sum of 4 + 1 or 1 + 4, but not 4 + 0 or 3 + 4, is the number after *four*—*five*).

Learning reasoning strategies plays a critical role in the meaningful memorization of combinations (Phase 3: an efficient, appropriate, and adaptive retrieval network) in two ways. One is that, with practice, reasoning strategies can become *automatic* (efficient and non-conscious; Jerman, 1970) and serve as a component of the retrieval system (Fayol & Thevenot, 2012). For instance, knowledge of the add-1 rule can be used to efficiently deduce any *n* + 1 or 1 + *n* combination, even previously unpracticed or multi-digit items, for which the child knows a number-after relation. The other way learning reasoning strategies can aid in achieving Phase 3 is that they provide children with an organizing framework for learning and storing both practiced and unpracticed combinations (Canobi, Reeve, & Pattison, 1998; Dowker, 2009; Rathmell, 1978; Sarama & Clements, 2009).

Most basic reasoning strategies

Research indicates that the easiest sums for children to learn are the add-1 and doubles families (see reviews by Brownell, 1941; Cowan, 2003). Given the informal knowledge children bring to school, the add-1 family is a developmentally appropriate (as well as logical) place to begin mental-addition training. At the start of school, most pupils are so familiar with the count sequence they can fluently specify the number after a given number (Fuson, 1988, 1992). Achieving fluency with adding 1 simply entails connecting it to their extant number-after knowledge—that is, recognizing the add-1 rule (Baroody, 1989, 1992; Baroody, Eiland, Purpura, & Reid, 2012; Baroody, Eiland, Purpura, & Reid, 2013).

The doubles are also relatively easy to learn because they embody familiar real-world pairs of a set, such as a dog's two front legs and two back legs make four legs altogether (Baroody & Coslick, 1998; Rathmell, 1978). Using a familiar everyday situation to determine the sum of a double involves analogical reasoning, the simplest and most common method of reasoning. For example, if a carton of a dozen eggs has 12 eggs and each of the two rows of six eggs is analogous to 6 + 6, then the sum of 6 + 6 is 12 also. Another reason learning the doubles is relatively easy is that it can build on several common aspects of primary-level mathematics instruction (Baroody & Coslick, 1998). One is that the sums of doubles are all even numbers and parallel the even number (skip-count-by-two) sequence: "2, 4, 6, . . ." Another aspect is that the sum of a double is akin to the first two counts in various skip counts (e.g., 5 + 5 = 10 can be reinforced by knowing the skip-count-by fives: "five, ten")—common aspects of primary-level mathematics instruction.

The add-1 and doubles combination families are the basis for more advanced mental-addition reasoning strategies. For example, efficiently implementing the make-10 (e.g., 9 + 5 = 9 + 1 + 4 = 10 + 4 = 14) and near-doubles strategies (e.g., 5 + 6 = 5 + 5 + 1 = 10 + 1 = 11) requires fluency with the add-1 rule. Note that the near-doubles strategy also requires fluency with the doubles, such as 5 + 5 = 10.

Instructional method of the interventions: why guided discovery learning?

In an extensive review of the literature, the NRC (2001) concluded that Phase 2 can be accelerated by directly teaching reasoning strategies, *if* done conceptually. Direct teaching of reasoning strategies accompanied by explanation of their rationale is often recommended by mathematics educators (Rathmell, 1978; Thornton, 1978, 1990; Thornton & Smith, 1988) and utilized in many elementary curricula, such as *Everyday Mathematics* (University of Chicago School Mathematics Project [UCSMP], 2005).

However, not all conceptually based instruction is equally effective (Baroody, 2003). Chi (2009) hypothesized that constructive activities (producing responses that entail ideas that go beyond provided information) are more effective than active activities (doing something physically), which in turn are more effective than passive activities (e.g., listening or watching without using, exploring, or reflecting on the presented material). Direct instruction—even when it attempts to illuminate the rationale for a reasoning strategy—typically embodies passive activities. As a result, it may not actively engage many children, be comprehensible, or produce routine expertise, which leads to applying a strategy inflexibly and inappropriately (Hatano, 2003). For example, Murata (2004) found that Japanese children taught a decomposition strategy with larger-addend-first combinations did not exhibit strategy transfer when smaller-addend-first items were introduced. Torbeyns, Verschaffel, and Ghesquiere (2005) found that children taught the near-doubles strategy sometimes used the strategy accurately but other times inaccurately (e.g., relating 7 + 8 to 7 + 7 – 1 or 8 + 8 + 1 instead of 7 + 7 + 1 or 8 + 8 – 1).

Discovery learning may be better suited to learning basic reasoning strategies than direct instruction because it can involve active learning and constructive activities (Swenson, 1949; Thiele, 1938; Wilburn, 1949). Alfieri, Brooks, Aldrich, and Tenenbaum (2011) defined discovery learning as not providing learners with the target information or conceptual information but creating the opportunity to "find it independently . . . with only the provided materials" (p. 2). Discovery learning encompasses a wide range of methods, which may not be equally effective in all cases. At one extreme is *highly guided discovery*—well-structured and moderately explicit instruction and practice. Although a pattern, relation, or strategy is not explicitly provided or explained to a child (as in direct instruction), this type of discovery learning involves considerable and explicit scaffolding. Instruction and practice are organized to direct a child's attention to regularities or a strategy. For example, items are arranged sequentially to underscore a pattern or relation and prompts direct attention toward a regularity or strategy without explicitly stating it. Feedback provides some explanation of *why* a response is correct or incorrect as well as specifying whether an answer is correct or not. At the other extreme is *unguided discovery*—unstructured and—sometimes called "free play." With this type of discovery learning, children chose their own task and do not receive adult feedback.

Research discussed in the present paper also involved two intermediate forms of discovery learning. *Moderately guided discovery* involves modest and implicit scaffolding, such as sequentially arranged items to underscore a relation so as to prompt its implicit recognition and feedback on correctness only. *Minimally guided discovery* entails a teacher-chosen task with little, implicit scaffolding, such as encouraging children to play a game involving a number list. Such programs described herein involved no verbal or written hints regarding relations problems, presenting items in a semi-random order (haphazard order except that related items were not presented together), and feedback that focused on correctness only.

Prior efforts

The initial software programs developed and evaluated by the authors involved minimally or moderately guided discovery learning of the add-1 rule and doubles tactics. Although these approaches appeared to contradict recent research reviews (Alfieri et al., 2011; Clark, Kirschner, & Sweller, 2012; Kirschner, Sweller, & Clark, 2006), there are three reasons to believe that a minimally

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