Artificial Intelligence in Medicine 65 (2015) 229–238

Contents lists available at ScienceDirect

Artificial Intelligence in Medicine

journal homepage: www.elsevier.com/locate/aiim

Sparse deconvolution of higher order tensor for fiber orientation distribution estimation

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ARTICLE INFO

Article history:
Received 7 January 2015
Received in revised form 10 August 2015
Accepted 2 September 2015

Keywords:
Diffusion magnetic resonance imaging
Fiber orientation distribution
Higher order tensor
Spherical deconvolution
Sparse approximation

ABSTRACT

Purpose: Higher order tensor (HOT) imaging approaches based on the spherical deconvolution framework have attracted much interest for their effectiveness in estimating fiber orientation distribution (FOD). However, sparse regularization techniques are still needed to obtain stable FOD in solving the deconvolution problem, particularly in very high orders. Our goal is to adequately characterize the actual sparsity lying in the FOD domain to develop accurate estimation approach for fiber orientation in HOT framework.
Materials and methods: We propose a sparse HOT regularization model by enforcing the sparse constraint directly on the representation of FOD instead of imposing it on coefficients of basis function. Then, we incorporate both the stabilizing effect of the l_2 penalty and the sparsity encouraging effect of the l_1 penalty in the sparse model to adequately characterize the actual sparsity lying in the FOD domain. Furthermore, a weighted regularization scheme is developed to iteratively solve the deconvolution problem. The deconvolution technique is compared against existing methods using l_2 or l_1 regularizer and tested on synthetic data and real human brain.
Results: Experiments were conducted on synthetic data and real human brain data. The synthetic experimental results indicate that crossing fibers are more easily detected and the angular resolution limit is improved by our method by approximately 20°–30° compared to existing HOT method. The detection accuracy is considerably improved compared with that of spherical deconvolution approaches using the l_2 regularizer and the reweighted l_1 scheme.
Conclusions: Results of testing the deconvolution technique demonstrate that it allows HOTs to obtain increasingly clean and sharp FOD, which in turn significantly increases the angular resolution of current HOT methods. With sparsity on FOD domain, this method efficiently improves the ability of HOT in resolving crossing fibers.

1. Introduction

Diffusion weighted magnetic resonance imaging (DW-MRI) is a non-invasive imaging technique capable of revealing microstructural information of human brain. Diffusion tensor imaging (DTI) is commonly used to approximate the diffusivity function from a given set of acquired DW-MRI images [1]. However, voxels are often contaminated with significant partial volume effects at current spatial resolutions, and the diffusion tensor model fails to accurately represent the nerve bundle geometry in voxels containing multiple fiber populations.

The need of developing accurate estimation approaches for fiber orientation to resolve neural architecture in regions with complex fiber patterns necessitates the inventing or using novel methods of high angular resolution diffusion imaging (HARDI). One group of methods, collectively known as q-space techniques, identifies multiple fiber components by calculating the probability distribution function (PDF) of the diffusion process based on the Fourier transform relationship between the PDF of the diffusion displacement and the diffusion weighted signal attenuation in q-space. These methods include Q-ball imaging [2], diffusion spectrum imaging [3,4], and diffusion orientation transform [5]. Model-based methods rely on a more complex model to characterize the diffusion-weighted signal attenuation; such methods include the multi-tensor model [6], directional functions [7], spherical harmonics deconvolution [8–12], and high order tensor (HOT)

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http://dx.doi.org/10.1016/j.artmed.2015.09.004
0933-3657/

[13–17]. Among these techniques, HOTs, which generalizes the 2nd order tensors, were introduced [13] to represent the non-Gaussian diffusion process owing to its simple polynomial form and its ability to model multi-lobed spherical functions. However, choosing a low-order basis implies an increased smoothing and therefore a loss of sharp directional information, whereas a high-order basis entails many more coefficients to represent the orientation distribution function (ODF). These coefficients, particularly those for the 8th, 10th or higher orders, are difficult to identify in diffusion estimation processing. Thus, the estimation of these coefficients suffers heavily from noise.

A common approach to robustly obtain the fiber orientation distribution (FOD) is to guarantee the non-negative property of HOT functions. Barmpoutis et al. [18] presented a novel technique that represents a 4th-order tensor using a homogeneous polynomial of degree 4 in three variables. The theories guaranteed the symmetric positive definite property. In succeeding works, they extended their method to represent a tensor of any order using homogeneous polynomial parameterizations that covers the full space of positive definite tensors of any order [16,19]; they summarized their work as a complete mathematical study [14]. Qi et al. [20,21] proposed a positive semi-definite diffusion tensor model that is a convex optimization problem with a convex quadratic objective function constrained by the non-negative requirement on the smallest Z-eigenvalue of the diffusivity function. Their research improved the stabilities, but their results are still prone to poor angular resolution.

Jiao et al. [15] recently expressed the spherical deconvolution of HOT-ODF as a linear programming problem and then extracted the fiber orientation using the rank-k tensor decomposition method. Weldeselassie et al. [16] used a spherical deconvolution framework in estimating the positive-definite Cartesian tensor ODF, which can be achieved by minimizing an objective function subject to nonnegative constraints. Such minimization is in turn achieved by solving a linear programming problem using the nonnegative least squares algorithm. These spherical deconvolution approaches represent a huge advance in obtaining increasingly sharp fiber orientation. However, the former method is known to suffer heavily from intrinsic instabilities in solving the non-negative linear programming problem. The latter method approximates the HOTs using the sum of squares of lower order tensors. The accuracy of the approximation also depends on how well the set of vectors is sampled in the space of a unit sphere.

Recent research on HOT is generally based on the assumption that FOD is a non-negative function. Incorporating prior knowledge on FOD via sparse representation provides an effective approach to reconstruct sparse fiber signals. Landman et al. [22] and Pu et al. [23] imposed the l_1-norm penalty on the coefficients of spherical harmonics basis. Daducci et al. [12] introduced l_0-norm penalty and then performed approximations through a reweighing l_1-norm scheme. These approaches assume that the sparsity of the coefficients implies the FOD sparsity. Hence, they usually directly enforce the FOD sparsity on the domain of coefficients. The FOD values are usually computed from a set of basis functions or polynomials with the coefficients. The sparsity of the coefficients means that only a few basis functions or polynomials are included to form the FOD, whereas the sparsity of the FOD suggests that only a small number of the FOD values are non-zero.

In this work, we first define a new sparse HOT imaging model, in which the sparse constraint is directly imposed on the FOD domain using the l_0-norm penalty. The l_0-norm sparsity of FOD is then approximated by combining the sparsity encouraging effect of l_1-norm and the stabilizing effect of l_2-norm. In addition, a weighted regularization scheme is proposed to iteratively solve the minimization problem. We evaluate the effectiveness of our proposal for improving FOD reconstructions by comparing it with existing state-of-the-art HOT-ODF and spherical harmonics methods. We report results on both synthetic and real data. This paper is organized as follows: Section 2 develops the sparse HOT imaging model that performs convolution between the HOT-ODF and DW-MRI signals and proposes the iterative deconvolution algorithm. Section 3 presents the evaluation of the performance of the algorithm on synthetic data and real-world DW-MRI data. Section 4 concludes the paper.

2. Approach

2.1. HOT fitting model for diffusion estimation

The image contrast in diffusion weighted imaging is related to the diffusion of water molecules, whose measurements can be rendered sensitive to water diffusion along distinct spatial direction g in the sphere, such that the diffusion signal attenuation $S(g)/S_0$ is measured with diffusion weighted b-values b for each direction. We express the diffusion signal attenuation profile as the convolution of a signal fiber response function using HOT-ODF

$$S(g)/S_0 = R(v, g) \otimes D(v) = \int_{S_2} R(v, g)D(v)dv \tag{1}$$

where $g = (g_1, g_2, g_3)^T$ is the magnetic field gradient direction, and v is the unit vector on sphere S^2. $S(g)$ represents the DW-MRI signal acquired with b-value b in direction $g \in S^2$, while S_0 is the signal acquired without diffusion weighting. $R(v, g)$ is an axially symmetric response function representing diffusion signal attenuation measured from a single coherently oriented fiber population[24], written as

$$R(v, g) = \exp^{-\mu b (g^T v)^2} \tag{2}$$

where $\mu = (\lambda_{\max} - \bar{\lambda})$ denotes anisotropy interaction to signal attenuation which can be estimated from the signal attenuation in areas of a single z-aligned rotationally symmetric fiber population with the eigenvalues $[\lambda_{\max}, \bar{\lambda}, \bar{\lambda}]$. The HOT-ODF function $D(v)$ is approximated using a Cartesian tensor [16]. For the lth order tensor, it reads

$$D(v) = \sum_{r=0}^{l} \sum_{s=0}^{l-r} d_{rs} v_1^r v_2^s v_3^{l-r-s} \tag{3}$$

where $v = (v_1, v_2, v_3)^T$ is the 3-dimensional unit vector, and d_{rs} are the tensor coefficients. For convenience, Eq. (3) can also be expressed as

$$D(v, x) = \sum_{j=1}^{m} x_j f_j(v) \tag{4}$$

where x_j are the coefficients related to the tensor coefficients d_{rs}, and $f_j(v) = v_1^r v_2^s v_3^{l-r-s}$ are the jth tensor monomials with $j = r(l - \frac{r-3}{2}) + s + 1$. The number of terms in the summation in Eq. (4) is bounded by the number of unique monomials, that is, $m \le (l+1)(l+2)/2$.

Given a data set of DW-MRI signal attenuations $S(g_i)/S_0$ associated with magnetic gradient orientations g_i and diffusion weighted b-values b, the coefficients of an lth order tensor can be estimated