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Intuitionistic fuzzy variables: Concepts and applications in decision making

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ABSTRACT

The present paper aims at advancing the well-recognized uncertainty theory built upon the credibility measure. In face of the increasing level of uncertainty embedded within the real world problem solving, it is discovered that the fuzzy variables defined in the credibility space are far from adequate to reflect the hesitance incurred during decision making. Although more complex fuzzy quantities are introduced, such as the fuzzy possibility space based type-2 fuzzy variables, the area of intuitionistic fuzzy variables is still a largely unexplored territory. Therefore, a novel concept of intuitionistic fuzzy variable is introduced hereby as an attempt to extend the uncertainty theory, which possesses a decent self-dual property compared with the fuzzy set theory. Unlike the intuitionistic fuzzy set theory, which has been widely implemented in various application fields, the intuitionistic fuzzy variables have hardly been employed in practices, especially the decision making sector. To bridge the gap, three outranking methods are developed to help evaluate intuitionistic fuzzy variables. The sufficient conditions of the three methods in distinguishing different fuzzy variables are also proved in the present work. The benefits of ranking with the three methods include the minimum amount of defuzzification, which prevents the information loss to a significant extent. Furthermore, in order to validate the proposed methods, a numerical study is performed. And the consistency of the ranking outputs from the three proposed methods is analyzed afterward. Besides, several managerial insights are presented regarding the sensitivity analysis of each ranking method.

1. Introduction

Fuzzy set theory, ever since its introduction (Zadeh, 1965), has been successfully applied to solve both academic and practical problems with uncertainty. And these problems with ambiguous information could not be easily tackled by the traditional deterministic or probabilistic methods. Later, in order to extend the fuzzy set theory to image more complex situations in the real world suitably, intuitionistic fuzzy set is introduced (Atanassov, 1986). The new form of fuzzy set consists of two parts, namely the membership function and the nonmembership function. Which could more accurately exhibit the possibility of the fuzzy events contained in each set. Much like the introduction of the fuzzy set theory, the intuitionistic fuzzy set theory has attracted plenty talented researchers to work on its theoretical framework (Beliakov, Bustince, Goswami, et al., 2011; Bustince, 2000; De, Biswas, & Roy, 2000; Szmidt & Kacprzyk, 2000; 2001), as well as its broad range of applications in the real cases (Gong, Li, Zhou, et al., 2009; Intepe, Bozdag, & Koc, 2013; Kahraman, Cevik, Ates, et al., 2007; Wu & Liu, 2013).

Aside from analyzing the various structures of the fuzzy set and its derivatives, another group of scholars focus on the axiomatic foundation of the fuzzy set theory. And the second category of research originates from the nature of the possibility theory inherent with the fuzzy set theory (Zadeh, 1978). Several years before Zadeh's paper, Kaufmann (1975) investigated the fuzzy variable with its possibility distribution and membership function. Later on, Nahmias (1978) Q2 introduced the concepts of fuzzy variable based on the possibility measure. Recently, initiated by Liu and Liu (2002), the research of the fuzzy variables based on the credibility measure arises new challenges in the field of credibility theory. The primary difference between the fuzzy variables proposed by Liu and the ones by Nahmias is their axiomatic foundation. The credibility measure presented by Liu possesses the most wanted self-duality property. Which does not exist in the possibility measure based counterparts. In Liu's latest book (Liu, 2011), the credibility measure and its superiority are explained in detail.

Further steps are taken to advance the framework of the uncertainty theory proposed by Liu and Liu (2002). For instance, Zhao et al. (2007) introduced the random fuzzy renewal process based on fuzzy

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variables. To correct their work, Hong (2014) proposed a new version of Blackwell's theorem for fuzzy variables. Besides the theoretical extensions in the fundamentals of fuzzy variable domain, noticeable efforts have been spent in the area of type-2 fuzzy variables, such as Liu and Liu (2010), Qin, Liu, and Liu (2011), and Bai and Liu (2014), dedicated to unveil the basic arithmetics and critical values of type-2 fuzzy variables. Following Liu and Liu's work (2010), the type-2 fuzzy variables based on fuzzy possibility measure have been intensively applied in the field of transportation scheduling and optimizations (Kundu, Kar, & Maiti, 2014; Liu, Yang, Wang, et al., 2014; Yang, Zhou, & Gao, 2014). Clearly, the type-2 fuzzy variables appeared in the above list of literatures all originate from the fuzzy variables based on possibility measure. In comparison, the intuitionistic fuzzy variables introduced in the present paper are based on credibility measure. Furthermore, the type-2 fuzzy variable based methods used in those papers all implement a mapping from the fuzzy possibility space to the real set. On the other hand, the intuitionistic fuzzy variable based decision making approaches proposed in the current work rely purely on the dominance and distance related theory within the credibility space to obtain the ranking outputs. And the only mapping used is to linearly transform the intuitionistic fuzzy variables into fuzzy variables under credibility measure. Although it might be considered unusual to involve the discussion of type-2 fuzzy variables in the paper with respect to the intuitionistic fuzzy variables, a recent study (Own, 2009) shows that the intuitionistic fuzzy sets, as they are isomorphic to the interval valued fuzzy sets, could establish links with the type-2 fuzzy sets. While which are regarded as the generalizations of interval valued fuzzy sets.

In the literature, a large quantity of research works are dedicated to solve the multi-attribute decision making problems in fuzzy settings (Chen & Hwang, 1992; Pei, 2013a). As the intuitionistic fuzzy set theory gradually draws more attention, the intuitionistic fuzzy decision making becomes one promising research prospect (Pei, 2013b; Pei & Zheng, 2012). In order to adapt the existing ranking methods to that trend, Xu (2007) evolved the fuzzy aggregation operators to the intuitionistic fuzzy aggregation operators. And in the following year, Xu and Chen (2008) introduced the distance and similarity measures of the intuitionistic fuzzy sets. The latest trend of intuitionistic fuzzy decision making would be the applications of new intuitionistic fuzzy information aggregation operators (Tan, Jiang, Chen, et al., 2015) and new comprehensive fuzzy outranking methods (Chen, 2015); and another promising direction for intuitionistic fuzzy decision making would be the ever growing scale and complexity of the decision making problems themselves (Liu, Shen, Zhang, et al., 2015; Pei, 2013a). An excellent review paper concerning the recent development of the intuitionistic fuzzy decision making is composed by Xu and Cai (2010). However, the decision making problems with fuzzy variables of credibility measure are not well developed (Li & Liu, 2008a). Peng, Mok, and Tse (2005) and Peng, Liu, and Shang (2006) introduced the concepts of fundamental fuzzy dominance relationship, and constructed one ranking method based on the credibility measure. Zeng (2006) proposed a fuzzy multi-attribute decision making method based on the expectations of fuzzy variables. In depth analysis about the expected value of the fuzzy variables with a continuous membership function is performed by Xue, Tang, and Zhao (2008). More recently, the distance based metric space of fuzzy variables is examined, and a miniature numerical example is listed (Tang, Li, & Zhao, 2009).

In the mean time, there are still limited studies of the intuitionistic fuzzy variables defined on possibility space from the perspective of statistics analysis, the most recent of which is Zainali, Akbari, and Noughabi (2014). Also, intuitionistic linguistic variables (Liu & Jin, 2012) and type-2 linguistic variables (Abdullah & Najib, 2014) are integrated in the decision making process. Where the linguistic variables are not built within the framework of possibility space, but the subscripts of their respective representations.

Nevertheless, to the best of our knowledge, there is hardly any research work falling into the scope of credibility measure based intuitionistic fuzzy variables, not to mention the multi-attribute decision making methods built upon such intuitionistic fuzzy variables. To bridge the gap, the present paper is devoted to introduce the fundamental concepts of a type of new fuzzy variables under the credibility space, and to implement the new fuzzy variables into multi-attribute decision making. Since these newly proposed fuzzy variables contain a membership part and a nonmembership segment, they are simply entitled as intuitionistic fuzzy variables. To render the newly defined fuzzy variables more beneficial to the decision making, and also to avoid information loss during defuzzification, a more complete fuzzy dominance approach and two distance based outranking methods are demonstrated. Apparently, these designed methods should meet one of the most important requirements of outranking, which is to bestow a unique ranking position to each alternative in their final order. Furthermore, a typical electrical heating-device manufacturer in China is employed as an empirical study to validate our speculations.

The content of the present paper is organized as below. In the second section, several preliminary knowledge with respect to the credibility theory and fuzzy variables are briefly stated. Section 3 introduces the conceptions of intuitionistic fuzzy variable, and presents a rather straightforward way to compare them with each other. In Section 4, the fuzzy dominance method and two distance based approaches are proposed to rank fuzzy variables. An actual heating-device manufacturer case is investigated in Section 5, then the generated outputs are analyzed in detail. Section 6 concludes the paper.

2. Preliminaries

In this section, the preliminary concepts of credibility space, fuzzy variable and intuitionistic fuzzy sets are introduced so as to facilitate further discussion.

Definition 2.1 (Fuzzy measure (Zadeh, 1978))**.** For the universe of discourse X, and the set $\mathcal{C}$ of its unempty subsets, the fuzzy measure in $(X, \mathcal{C})$ is defined as the function $g : \mathcal{C} \to [0, 1]$, which meets the following conditions.

(1) Boundary condition: $g\{\phi\} = 0, g\{X\} = 1$;
(2) Monotonicity: for $A, B \in \mathcal{C}$, if $A \subseteq B$, then $g\{A\} \leq g\{B\}$.

In order to measure the fuzzy events, Zadeh introduced the possibility measure and necessity measure in 1978 and 1979, respectively.

Definition 2.2 (Possibility measure (Zadeh, 1978))**.** Suppose Θ is an unempty set, $\mathcal{P}(\Theta)$ is the power set of Θ, and the function $\mathrm{Pos} : \mathcal{P}(\Theta) \to [0, +\infty]$ is defined as the possibility measure, when it satisfies the following three axioms.

Axiom 1: $\mathrm{Pos}\{\Theta\} = 1$;
Axiom 2: $\mathrm{Pos}\{\phi\} = 0$;
Axiom 3: for any set of subsets $\{A_i\}$ in $\mathcal{P}(\Theta)$, the following equation holds:

$$\mathrm{Pos}\{\bigcup_i A_i\} = \sup_i \mathrm{Pos}\{A_i\}.$$

The possibility measure $\mathrm{Pos}\{A\}$ indicates the possibility of the fuzzy event A.

Definition 2.3 (Possibility space)**.** Suppose Θ is an unempty set, $\mathcal{P}(\Theta)$ is the power set of Θ. If Pos is the possibility measure, then the triple $(\Theta, \mathcal{P}(\Theta), \mathrm{Pos})$ is defined as the possibility space.

The possibility space has three fundamental attributes as below:

(1) Boundary condition: For any $A \in \mathcal{P}(\Theta)$, $0 \leq \mathrm{Pos}\{A\} \leq 1$ holds;
(2) Monotonicity: If $A, B \in \mathcal{P}(\Theta)$, we have $A \subseteq B$, then $\mathrm{Pos}\{A\} \leq \mathrm{Pos}\{B\}$;

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