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Latent space robust subspace segmentation based on low-rank and locality constraints

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ABSTRACT

Low-rank representation (LRR) and its extensions have proven to be effective methods to handle different kinds of subspace segmentation applications. In this paper, we propose a new subspace segmentation algorithm, termed latent space robust subspace segmentation based on low-rank and locality constraints (LSRS2). Different from LRR, LSRS2 learns a low-dimensional space and a coefficient matrix for a data set simultaneously. In the obtained latent space, the coefficient matrix can faithfully reveal both the global and local structures for the data set. Furthermore, we build the connections between LSRS2 and robust coding methods, and show LSRS2 can be regarded as a kind of robust LRR method. Therefore, it can be guaranteed in theory that LSRS2 shows good performance. In addition, an efficient optimization method for solving LSRS2 is presented and its convergence is also proven. Extensive experiments show that the proposed algorithm outperforms the related subspace segmentation methods.

1. Introduction

In computer vision and patter recognition fields, it is one of the most challenging problems to recover inherent structures for high dimensional data. In most cases, high dimensional data from different classes can be viewed as samples generated from a union of linear subspaces (Liu et al., 2013; Rao, Tron, Vidal, & Ma, 2010). Hence, many kinds of machine learning algorithms such as Gaussian mixture model (Bishop, 2007; He, Cai, Shao, Bao, & Han, 2011; Ma, Yang, Derksen, & Fossum, 2008), matrix factorization (Costeira & Kanade, 1998; Lee & Seung, 1999), similarity-based methods (Duda, Hart, & Stork, 2000; Ng, Jordan, & Weiss, 2001; Shi & Malik, 2000) and representation coefficient based methods (Elhamifar & Vidal, 2009; Hu, Lin, Feng, & Zhou, 2014; Liu et al., 2013) have been proposed to tackle the subspace segmentation problems.

Gaussian mixture model assumes that mixed data is drawn from a mixture of Gaussian distributions. By using the expectation maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977) to solve a model estimation problem, the obtained conditional probabilities of data points can be used to segment the data. The main drawback of Gaussian mixture model based approaches is that they are sensitive to the noise and outliers (Liu, Lin, & Yu, 2010).

Factorization-based methods (Costeira & Kanade, 1998; Lee & Seung, 1999) seek two matrices (factors) whose product provides a good approximation to the original data matrix. Then one of the two matrices actually discovers the segmentation of data samples. When the data is grossly corrupted, it can be found that algorithms used to solve factorization-based models will be usually trapped at local minima. K-means (Ding & He, 2004; Duda et al., 2000) and spectral clustering (Ng et al., 2001; Shi & Malik, 2000) are two kinds of representative algorithms of similarity-based methods. Because of the great successes achieved in image segmentation (Shi & Malik, 2000) and many other real world applications (Cai, He, Ma, Wen, & Zhang, 2004; Higham, Kalna, & Kibble, 2007), spectral clustering algorithms attract lots of researchers' interests. However, the performance of spectral clustering largely depends on whether or not the constructed affinity graphs can reveal the structures of data sets. Traditional spectral clustering algorithms usually use K-nearest-neighbor (KNN) (Duda et al., 2000) method to construct affinity graphs. But KNN may not be capable of discovering the intrinsic structures for different kinds of complex data sets.

Representation coefficient based methods could be regarded as the extensions of spectral clustering. For a group of data samples, they use the data set itself to represent each data point. Then the obtained representation coefficient vectors are concentrated to form an affinity graph. Finally, a spectral clustering algorithm such as Normalized Cut (N-cut) (Shi & Malik, 2000) is used to obtained the segmentation solution. The most well-known representation

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coefficient based subspace segmentation methods are sparse subspace clustering (SSC) (Elhamifar & Vidal, 2009) and low-rank representation (LRR) (Liu et al., 2010, 2013). Elhamifar and Vidal proposed SSC based on sparse representation (SR) (Wright, Yang, Ganesh, Sastry, & Ma, 2009) method. Liu et al. argued that SR computes the sparse representation of each data individually, so the l_1-norm graphs used in SSC may not be able to discover the data's global structures and the performance of SSC will be reduced when data is grossly corrupted (Liu et al., 2010; Liu et al., 2013). Consequently, they devised a low-rank representation (LRR) algorithm which finds the lowest-rank representations of all data jointly. Compared with SSC, LRR achieved much better subspace segmentation results on different kinds of data sets. Inspired by LRR, Zhuang et al. added sparse and non-negative constraints on the coefficient matrix and presented a nonnegative lowest-rank and sparsest representation method (NNLRSR) (Zhuang et al., 2012). Tang et al. performed a deeper analysis on NNLRSR and proposed the structure-constrained low-rank representation (SCLRR) (Tang, Liu, & Zhang, 2014). They claimed that NNLRSR was actually a special case of SCLRR. Li et al. added a b-match constraint into LRR to construct balance affinity graphs for semi-supervised learning (Li & Fu, 2013). By incorporating the label information of data sets, Zhang et al. devised a structured low-rank representation (SLRR) method and a supervised learning method to construct a discriminative dictionary based on SLRR (Zhang, Jiang, & Davis, 2013). Chen et al. proposed a discriminative low-rank representation (DLRR) (Chen & Zhang, 2014) which compelled the distances between the low-rank representations of data samples in different classes to be as large as possible. Liu et al. proposed a latent low-rank representation (LatLRR) (Liu & Yan, 2011) method which could use both observed data and hidden data to resolve insufficient sampling problems existed in LRR. Zhang et al. claimed that the low-rank representations of nearby data points should be similar to each other. Hence, they proposed a regularized low-rank representation (rLRR) framework (Zhang, Yan, & Zhao, 2014) by adding a Laplacian regularization term into the objective function of LatLRR. Hu et al. analysed the group effect of the existing representation methods and proposed a kind of smooth representation clustering algorithm (Hu et al., 2014). These algorithms achieved the state-of-the-art results in different kinds of subspace segmentation tasks.

On the other hand, feature extraction or dimensionality reduction (Belhumeur, Hespanha, & Kriegman, 1997; He, Yan, Hu, Niyogi, & Zhang, 2005; Martinez & Avinash, 2001; Yan et al., 2007) is a necessary preliminary for practical data processing. Feature extraction algorithms aim to find low-dimensional subspaces in which the structures of data sets can be easily recovered. Hence, it becomes a natural idea to combine feature extraction algorithms and representation coefficient based subspace segmentation algorithms together. Recently, Patel et al. proposed an extension of SSC, termed latent space sparse subspace clustering (LS3C) algorithm (Patel, Nguyen, & Vidal, 2013). For a data set, LS3C learns a low-dimensional space and a sparse coefficient matrix simultaneously. In the obtained low-dimensional space, SSC can achieve much better results.

Though the experiments show that LS3C outperforms SSC, the theoretical explanation has not been provided. In this paper, we would show the relationships between LS3C and robust coding methods (He, Zheng, & Hu, 2011; Lai, Dai, Ren, & Huang, 2014; Yang, Zhang, Yang, & Zhang, 2011). Based on our analyses, it can be seen that LS3C is a special case of robust sparse coding methods. This can explain the reason why LS3C is superior to SSC. However, similar to SSC, l_1-norm graphs obtained by LS3C are still insufficient to characterize the global structures of data sets, we therefore propose a new latent space learning method for subspace segmentation based on low-rank constraint. According to the analyses on robust coding methods, the proposed algorithm can be viewed as a kind of robust low-rank representation method (Chen & Yang, 2014). Moreover, though low-rank constraint is helpful to reveal the global structures of data sets, local structures are also useful in subspace segmentation (He et al., 2011). Hence, we devise a weighted l_1-norm regularizer in the objective function of our proposed algorithm to capture the local structures. Thus our algorithm is termed latent space robust subspace segmentation based on low-rank and locality constraints (LSRS2). The experiments conducted on both synthetic and real data sets prove the conclusion that LSRS2 is superior to LS3C and the related representation coefficient based subspace segmentation algorithms. The contributions of this paper can be summarized as follows:

1. We build the connections between LS3C and robust coding methods, consequently explain the success of LS3C;
2. From the point of view of robust low-rank representation, we put forward a new latent space robust subspace segmentation method based on low-rank constraint;
3. We devise a weighted l_1-norm regularizer to help the proposed algorithm to capture the local structures of data sets.

The rest of this paper is organized as follows: Section 2 briefly reviews SSC and LS3C algorithms. Section 3 introduces the ideas of robust coding methods. And we will show the connections between LS3C and robust coding methods. Following the analyses presented in the Section 3, we propose the latent space robust subspace segmentation based on low-rank and locality constraints (LSRS2) algorithm in Section 4. We also describe the differences between LSRS2 and the closely related algorithms. In addition, the optimization approach and the proof for its convergence are presented in this section. The extensive experiments performed to show the effectiveness of LSRS2 are presented in Section 5. Finally Section 6 gives the conclusions.

2. Latent space sparse subspace clustering (LS3C)

In this section, we will review LS3C briefly. LS3C algorithm is devised under the framework of SSC. We first introduce SSC method.

2.1. Sparse subspace clustering (SSC)

SSC uses SR to construct the affinity graph for a group of data. Suppose the data set $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n] \in R^{D\times n}$. According to the SR theory, each sample $\mathbf{x}_i \in \mathbf{X}$ can be linearly represented by using as few as possible the rest samples in $\mathbf{X}$. This purpose can be expressed as follows:

$$\min_{\mathbf{z}_i} \|\mathbf{x}_i - \mathbf{X}\mathbf{z}_i\|_2^2 + \lambda\|\mathbf{z}_i\|_1 \quad s.t. \quad z_{ii} = 0, \tag{1}$$

where $\mathbf{z}_i$ is the coefficient vector which also reveals the relationship between $\mathbf{x}_i$ and the rest samples, z_{ii} is the ith element of $\mathbf{z}_i$. Eq. (1) is also called LASSO[1] (Tibshirani, 1996). $\|\mathbf{z}_i\|_1 = \sum_{j=1}^{n}|z_{ij}|$ is the l_1-norm of $\mathbf{z}_i$. Considering all the data points in $\mathbf{X}$, we can get the following optimization problem:

$$\min_{\mathbf{Z}} \|\mathbf{X} - \mathbf{X}\mathbf{Z}\|_F^2 + \lambda\|\mathbf{Z}\|_1 \quad s.t. \quad diag(\mathbf{Z}) = \mathbf{0}, \tag{2}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. For a certain matrix $\mathbf{A} \in R^{m\times n}$, its Frobenius norm $\|\mathbf{A}\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}[\mathbf{A}]_{ij}^2}$, $[\mathbf{A}]_{ij}$ represents

[1] A typical LASSO problem could be expressed as $\min_{\mathbf{z}}\|\mathbf{x} - \mathbf{D}\mathbf{z}\|_2^2 + \lambda\|\mathbf{z}\|_1$, where $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \cdots, \mathbf{d}_m]$ is the dictionary, $\mathbf{d}_i$ is the ith column of $\mathbf{D}, \mathbf{z}$ is the coefficient vector.