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An influence-knot set based new local refinement algorithm for T-spline surfaces

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ABSTRACT

Local refinement is widely applied in surface modeling. This paper proposes the concept of influence knot set, and uses it to establish a new algorithm for the local refinement of T-spline surfaces, whose effectiveness is demonstrated by several examples. Compared with existing algorithms, the present one may have two advantages: (a) it does not produce excessive propagation of control points because the number of additional control points is reduced; (b) it simplifies the process of judging whether an additional control point needs to be added or not, and so the final T-mesh topology can be obtained more easily.

1. Introduction

Piecewise polynomials such as spline functions have important applications in approximation theory and numerical analysis. With the development of computer graphics, they are widely used in function approximation and data fitting. Since 1980s, NURBS (Non-Uniform Rational B-Spline) has been an important conventional tool for describing the shapes of curves and surfaces, and it has been widely applied in the shape design of airplanes, cars, ships and other products (Cohen, Riesenfeld, & Elber, 2001; Piegl & Tiller, 1997).

NURBS is able to model free surfaces and represent all quadric surfaces accurately, and has good mathematical properties including continuity, variation diminishing, convex hull, and etc. However, one of its disadvantages is that it is quite difficult to perform local refinement on NURBS surfaces. In the process of local refinement, an entire row of control points have to be inserted because NURBS control points must lie topologically in a rectangular grid. Therefore, superfluous control points cannot be avoided, and excessive propagation of control points are always produced.

To overcome the disadvantages of NURBS, Sederberg et al. proposed T-spline surfaces (Sederberg, Zheng, Bakenov, & Nasri, 2003; Sederberg et al., 2004). Compared with other modeling methods, T-splines have many advantages in surface merging, local refinement, and data compression, etc. Due to these advantages, T-splines can be employed to construct complex surfaces. Therefore, they are desirable for use in digital design and manufacturing. Since the introduction of T-splines, many researchers have launched extensive investigations on the theories and applications of T-splines (Deng et al., 2008; Li, Zheng, Sederberg, Hughes, & Scott, 2012; Nguyen-Thanh, Nguyen-Xuan, Bordas, & Rabczuk, 2011b; Nguyen-Thanh et al., 2011a; Scott, Li, Sederberg, & Hughes, 2012; Wang & Zhao, 2011; Wang & Zhao, 2013). In 2012, Li et al. presented analysis-suitable T-splines (Li et al., 2012), whose blending functions are linearly independent. At the same time, Scott et al. gave a local refinement algorithm for analysis-suitable T-splines (Scott et al., 2012). NURBS control points must lie topologically in a rectangular mesh. Different from NURBS, the T-spline is defined by T-mesh, which allows extraordinary knots and T-junctions in their control mesh. In general, extraordinary knots often appear in the complicated geometries. Then, it is desirable to use T-splines to construct complex surfaces in some complicated geometries.

Local refinement is widely used in surface design. Unlike NURBS, T-splines can be locally refined without introducing a complex hierarchy of meshes (Forsey & Bartels, 1988; Sederberg et al., 2004). T-spline local refinement allows adding new control points to a surface without changing its shape. It has advantages in three aspects: (a) adding more control points can make the surfaces more flexible to be edited, which is convenient for further adjustment; (b) local refinement algorithm can be used to realize two-way conversion between T-spline and B-spline surfaces; (c) when more control points are added through local refinement, the control mesh becomes closer to the T-spline surface.

T-spline local refinement algorithm was first introduced in literature (Sederberg et al., 2003), according to which a new control point can be inserted into a T-mesh topology only if some additional control points are also added. However, the algorithm in literature (Sederberg et al., 2003) has two drawbacks: (a) The number of additional control points can be quite large; (b) for some particular T-spline topologies, it is impossible to insert a requested

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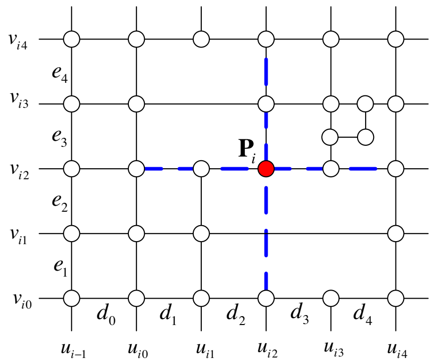

Fig. 1. A local region of a T-mesh.

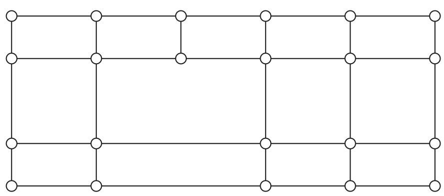

Fig. 2. A standard T-spline.

control point. To solve these problems, Sederberg et al. put forth an improved algorithm in literature (Sederberg et al., 2004), which makes it always possible to insert a requested control point. Later, this improved algorithm was applied in the software named *T-Spline for Rhino*. Local refinement generally includes two stages, the topology stage and the geometry one. In the topology stage, the improved algorithm in literature (Sederberg et al., 2004) can generate the final T-mesh topology only after adding control points through multiple iterations, which may reduce the efficiency of the local refinement. Therefore, it is not convenient to determine the location of additional control points from the initial T-mesh directly.

To solve these problems, this paper proposes a new T-spline refinement algorithm that enhances the refinement efficiency by simplifying the process of the topology stage. This paper is organized as follows. Section 2 reviews T-splines. Section 3 presents the new T-spline local refinement algorithm, which is our main work. Compared with literature (Sederberg et al., 2004), the present algorithm can simplify the process of judging whether an additional control point should be inserted or not, and can generate the final T-mesh faster. Examples are given in section 4 to verify the new algorithm. Finally, the concluding remarks are given in section 5.

2. T-splines

The equation of a T-spline surface is (Sederberg et al., 2004)

$$\mathbf{P}(u, v) = \sum_{i=1}^{n} \mathbf{P}_i B_i(u, v) \tag{1}$$

where $\mathbf{P}_i(i = 1,2,\ldots,n)$ are control points; $B_i(u,\ v)$ is the blending function corresponding to $\mathbf{P}_i$; $B_i(u,v) = N^3_{iu}(u)N^3_{iv}(v)$; $N^3_{iu}(u)$ and $N^3_{iv}(v)$ are B-spline basis functions associated with the knot vectors $\mathbf{u}_i = [u_{i0}, u_{i1}, u_{i2}, u_{i3}, u_{i4}]$ and $\mathbf{v}_i = [v_{i0}, v_{i1}, v_{i2}, v_{i3}, v_{i4}]$, respectively. For a T-spline surface, the knot information of blending functions is determined from the T-mesh which satisfies some corresponding constraint rules (Sederberg et al., 2003). The present paper is restricted to the case of three-degree T-splines, though the concepts can be extended to any degree.

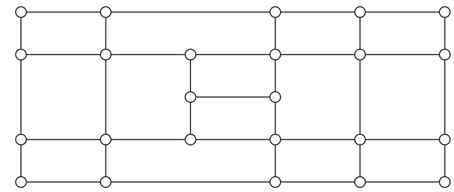

Fig. 3. A semi-standard T-spline.

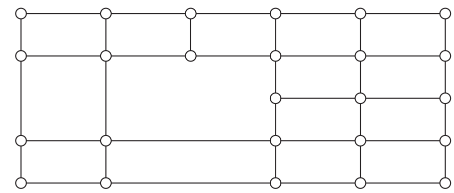

Fig. 4. A non-standard T-spline .

2.1. T-spline blending functions

For NURBS, all the control points must lie topologically at the vertexes of rectangular grids, i.e. the knot sequence of each row or column is the same. Different from this, intermediate control points may exist on the edges of the grids of a T-spline surface so that the knot sequence of each row or column can be different, as illustrated in Fig. 1. Therefore, NURBS can be treated as a special case of T-splines. Since extraordinary knots are involved, T-splines generally use local parametric coordinate systems to calculate the position information of additional control points. Let the knot coordinates of a T-spline control point $\mathbf{P}_i$ be (u_{i2}, v_{i2}). Unlike NURBS, the knot vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ of $B_i(u,v)$ are determined as follows

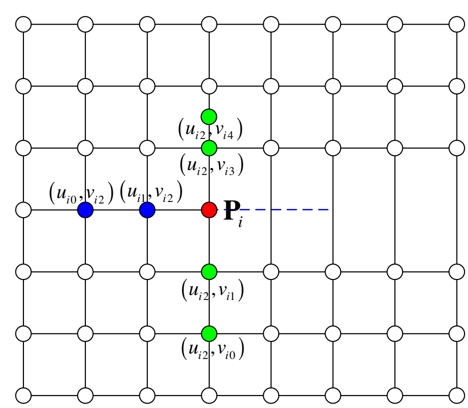

Fig. 5. The influence control point set of a control point.

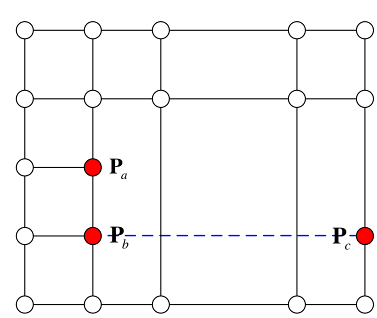

Fig. 6. Distance between two control points.