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Measuring the performance improvement of a double generally weighted moving average control chart

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ABSTRACT

This study extends the sum of squares generally weighted moving average (SS-GWMA) control chart by using the double generally weighted moving average (DGWMA) technique. The proposed expanded chart is called the sum of squares double generally weighted moving average (SS-DGWMA) control chart. Simulations are performed to evaluate the average run length (ARL) and standard deviation of run length (SDRL) of the SS-DGWMA, SS-DEWMA, and SS-GWMA charts. An extensive comparison shows that the optimal SS-DGWMA chart is superior to the optimal SS-GWMA and SS-DEWMA charts in all studied scenarios. The SS-DGWMA chart is also easy to implement and to interpret the abnormal signals.

1. Introduction

The major objectives of statistical process control (SPC) are to maintain the stability of a process and to detect the occurrence of assignable causes as early as possible. To achieve these objectives control charts are used to monitor the performance of a process and to improve the detection of abnormal process behavior.

The concept of a control chart was first proposed by Shewhart in the 1920s. Many researchers have since proposed various types of control charts to track the process variation in the mean or variance. Although the Shewhart-type chart remains the most popular chart in practice, it is not sensitive enough to detect small process shifts. Advanced process-monitoring techniques such as the exponentially weighted moving average (EWMA) or the cumulative sum (CUSUM) control charts have therefore been developed to compensate for the inefficiency of Shewhart control charts.

Roberts (1959) first introduced the EWMA control chart to control the process mean. Later, the EWMA control chart was used to monitor the process variance as well. Zhang and Chen (2005) extended the EWMA control chart to the double exponentially weighted moving average (DEWMA) control chart and proved that its detection ability was superior to that of the EWMA chart. To detect small shifts in the process mean or variability as early as possible, Sheu and Lin (2003) and Sheu and Tai (2006) developed and applied an expanded EWMA control chart called the generally weighted moving average (GWMA) control chart. Owing to an added adjustment parameter α, GWMA charts were found to be more sensitive than EWMA charts in detecting small process shifts. Furthermore, Sheu and Hsieh (2009) extended the GWMA chart to the double GWMA (DGWMA) chart. They also demonstrated that the DGWMA control chart with time-varying control limits was more sensitive than the GWMA and DEWMA control charts in detecting medium shifts in the process mean when the standard deviation of the shifts was between 0.5 and 1.5.

Traditionally, two control charts have been used to monitor the mean and variance of a process. One is used for the process mean and the other for the process variance. Commonly used combination-type charts are the combined Shewhart charts, combined EWMA charts, and combined GWMA charts. However, the use of such charts is time consuming and can potentially lead to increased costs. More recently, considerable attention has been focused on the use of a single chart to monitor both the process mean and variability. Domangue and Patch (1991) developed an omnibus EWMA chart for simultaneously detecting changes in both the location and spread of a process. Xie (1999) presented several different types of EWMA charts such as Max-EWMA, sum of squares EWMA (SS-EWMA), EWMA-Max, and EWMA semicircle (EWMA-SC). Chen, Cheng, and Xie (2001) and Chen, Cheng, and Xie (2004) extended the research of Xie on the Max-EWMA and the EWMA-SC charts. Costa and Rahim (2004) considered the joint monitoring of the process mean and variability using a non-central chi-square (NCS) chart. Based on the weighted loss function, Wu and Tina (2005) proposed a

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weighted loss function CUSUM (WLC) chart to monitor the changes in the mean and variance. An overview of single variable charts can be found in Chen and Thaga (2006). They mentioned that there were two approaches for using one control chart for process monitoring. One approach plots two quality characteristics in the same chart, whereas the other uses one plotting variable to represent the process location and spread. Costa and De Magalhaes (2007) proposed an adaptive non-central chi-square statistic chart to simultaneously monitor the process mean and variance. Wu, Wang, and Wang (2007) presented a loss-function-based adaptive control chart to monitor process shifts in the mean and variance simultaneously. Wu and Yu (2010) proposed a neural network ensemble model for online monitoring of process mean and variance shifts in correlated processes. The simulation results indicate that the model monitors any type of changes more efficiently than other traditional control charts, and can accurately classify the types of shifts for correlated processes. Khoo, Teh, and Wu (2010) revealed that the Max-DEWMA chart was more sensitive than the Max-EWMA chart for detecting small and moderate shifts in the process mean and/or variance. Teh, Khoo, and Wu (2011) proposed a SS-DEWMA chart to improve the performance of the SS-EWMA chart in detecting initial out-of-control signals. Sheu, Huang, and Hsu (2012) extended the single Max-EWMA chart to a single GWMA chart, called the Max-GWMA chart. Their analytical results indicated that the Max-GWMA chart was more sensitive than the Max-EWMA chart. Teh and khoo (2012) compared the effects of non-normality on the performances of Max-DEWMA and SS-DEWMA charts. They suggested that the Max-DEWMA chart be employed for the joint monitoring of the mean and/or variance, when the underlying distribution is non-normal. Ou, Wu, and Lee (2013) proposed an adaptive absolute cumulative sum chart (adaptive ACUSUM chart) for statistical process control. The new development includes the variable sampling interval (VSI), variable sample size (VSS), and VSS and interval (VSSI) versions, all of which are highly effective in monitoring the mean and variance of a variable by inspecting the absolute sample shift. Sheu, Huang, and Hsu (2013) presented a GWMA chart that detects both the mean and standard deviation shifts by inspecting a single MCSGWMA statistic. The comparison of the average run lengths (ARLs) shows that the MCSGWMA control chart performs better than the Max-EWMA control chart. Recently, the single SS-GWMA chart was proposed by Huang and Methods (2012). His simulation results indicated that SS-GWMA control charts outperformed SS-EWMA charts in terms of the ARL and standard deviation of run length (SDRL).

To enhance the detection ability of the SS-GWMA control chart, we employ the DGWMA technique developed by Sheu and Hsieh (2009) to detect the shifts in the process mean and/or process variability. This novel control chart is called the sum of squares DGWMA (SS-DGWMA) chart, and a statistical model is developed for the SS-DGWMA scheme. Numerical simulations are performed to calculate the ARLs and SDRLs of the SS-GWMA, SS-DEWMA, and SS-DGWMA charts. Based on the simulation results, we show that the optimal SS-DGWMA chart is superior to the optimal SS-GWMA and SS-DEWMA charts in all studied scenarios.

The rest of this article is organized as follows: Section 2 presents a review of the SS-GWMA chart. In Section 3, we introduce our proposed SS-DGWMA chart. Section 4 shows the implementations of the SS-DGWMA chart. The results of a simulation study are presented in Section 5, which compares the performance of the SS-DGWMA, SS-DEWMA, and SS-GWMA charts in terms of their initial-state ARL and SDRL. An illustrative example is presented in Section 6, whereas conclusions are drawn in Section 7. Finally, the technical details are provided in the Appendix.

2. Brief review of SS-GWMA control chart

The SS-GWMA chart was originally proposed by Huang (2012) to improve the performance of the SS-EWMA chart in detecting initial out-of-control signals. In this chart, two GWMA statistics are combined into a single chart and the process mean and variability can be simultaneously monitored.

A GWMA is a moving average of past data where each data point is assigned a weight. The SS-GWMA control chart is briefly introduced here for completeness.

In a sequence of independent samples, let M represent the number of samples until the recurrence of event A after a previous occurrence. For the sequence, we can write

$$\sum_{m=1}^{\infty} P(M=m) = P(M=1)+P(M=2)+\cdots+P(M=t)+P(M>t)=1. \tag{1}$$

Here, $P(M=1), P(M=2), \ldots, P(M=t)$ are the weights of the samples going backward from $M=1$ (the current sample) to $M=t$ (the remotest sample). Therefore, $P(M>t)$ is weighted with the target value of the process.

Let X be a quality characteristic of a process having a normal distribution with mean $\mu+\delta\sigma$ and standard deviation $\rho\sigma$, where μ and σ are defined as standard values of the process. If $\delta=0$ and $\rho=1$, the process is in control; otherwise, the process is shifted and/or changed.

Let X_{ij}, $i=1,2,\ldots$ and $j=1,2,\ldots,n_i$ be measurements of the variable X, arranged in groups of size n_i, where i is the index of the group number. Let $\bar{X}_i$ and S_i^2 denote the sample mean and sample variance, respectively, of sample i. Then, $\bar{X}_i$, $i=1,2,\ldots$ are independent normal random variables with mean $\mu+\delta\sigma$ and variance $\rho^2\sigma^2/n_i$; $(n_i-1)S_i^2/\rho^2\sigma^2$ and $i=1,2,\ldots$ are independent chi-square random variables with n_i-1 degrees of freedom, and $\bar{X}_i$ and S_i^2 are independent variables. Two statistics are defined:

$$U_i = \frac{\overline{X}_i-\mu}{\sigma/\sqrt{n_i}} \tag{2}$$

and

$$V_i = \Phi^{-1}\left\{F\left[\frac{(n_i-1)S_i^2}{\sigma^2},\ n_i-1\right]\right\}. \tag{3}$$

Note that in Eq. (3), $\Phi^{-1}(\cdot)$ and $F(h,\upsilon)$ denote the inverse standard normal distribution function and the chi-square distribution function with υ degrees of freedom, respectively (these transformations and their applications were proposed by Quesenberry (1995)).

Both U_i and V_i are independent standard normal random variables when the process is in control, and the distributions of both U_i and V_i are independent of the sample size n_i. Two GWMA statistics, one each for the mean and variability, can be defined from U_i and V_i as follows:

$$\begin{cases} A_i = P(M=1)U_i+P(M=2)U_{i-1}+\cdots+P(M=i)U_1+P(M>i)A_0 \\ A_0=0 \end{cases} \tag{4}$$

$$\begin{cases} B_i = P(M=1)V_i+P(M=2)V_{i-1}+\cdots+P(M=i)V_1+P(M>i)B_0 \\ B_0=0 \end{cases}, \tag{5}$$

where $i=1,2,\ldots$. The practitioner sets the starting value of A_0 and B_0, usually to $A_0=B_0=0$. It is known that A_i and B_i are independent because U_i and V_i are independent, and when $\delta=0$, $\rho=1$, and $A_0=B_0=0$, we have both $A_i \sim N(0,\sigma_{A_i}^2)$ and $B_i \sim N(0,\sigma_{B_i}^2)$, where $\sigma_{A_i}^2=\sigma_{B_i}^2=\sum_{j=1}^{i}[P(M=j)]^2$.