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Open vehicle routing problem with demand uncertainty and its robust strategies

Cao Erbao [a,b,*], Lai Mingyong [a,b], Yang Hongming [c]

[a] Department of Economics and Trade, Hunan University, Changsha 410079, China
[b] Hunan Province Laboratory of Logistics Information and Simulation Technology, Changsha 410079, China
[c] College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha 410004, China

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ABSTRACT

We investigate the open vehicle routing problem with uncertain demands, where the vehicles do not necessarily return to their original locations after delivering goods to customers. We firstly describe the customer's demand as specific bounded uncertainty sets with expected demand value and nominal value, and propose the robust optimization model that aim at minimizing transportation costs and unsatisfied demands in the specific bounded uncertainty sets. We propose four robust strategies to cope with the uncertain demand and an improved differential evolution algorithm (IDE) to solve the robust optimization model. Then we analyze the performance of four different robust strategies by considering the extra costs and unmet demand. Finally, the computational experiments indicate that the robust optimization greatly avoid unmet demand while incurring a small extra cost and the optimal return strategy is the best strategy by balancing the trade-off the cost and unmet demand among different robust strategies.

1. Introduction

OVRP was first introduced by Sariklis and Powell (2000). The OVRP is encountered in practice in many contexts, for example, the companies that outsource the deliveries to the third party logistics provider (3PL) as the external vehicles are not obligated to return to the depot (Cao & Lai, 2010; Lin, Choy, Ho, Chung, & Lam, 2013). The key feature of OVRP is that the vehicles do not necessarily return to their original locations after delivering goods to customers, and if they do, they must visit the same customers in the reverse order (Sariklis & Powell, 2000). The OVRP differs from the well-known vehicle routing problem (VRP) (Fleszar, Osman, & Hindi, 2009; Lin et al., 2013). The major difference in theory between the OVRP and the VRP (Tang, Ma, Guan, & Yan, 2013) is that the routes in the OVRP consist of Hamiltonian paths originating at the depot and terminating at one of the customers, while the routes in the VRP are Hamiltonian cycles (Fleszar et al., 2009; Fu, Eglese, & Li, 2005).

Solving OVRP is a scientific challenge in that it is a NP hard problem. The traditional studies of the OVRP assumed that the demands of all customers visited on its route by any vehicles were deterministic. Several researchers focused on heuristic or metaheuristic algorithms to solve the OVRP. Brandao (2004) and Fu et al. (2005) implemented tabu search (TS) heuristic to solve the OVRP with constraints on vehicle capacity and maximum route length. Li, Golden, and Wasil (2007) proposed a record-to-record travel heuristic and a deterministic variant of simulated annealing to solve the OVRP. Fleszar et al. (2009) presented a variable neighourhood search algorithm for the open vehicle routing problem. Li, Leung, and Tian (2012) propose a multistart adaptive memory programming metaheuristic with modified tabu search algorithm to solve a heterogeneous fixed fleet OVRP, in which the capacity and costs per unit distance of vehicles are heterogeneous. However, in the practical distribution operation, it is unreasonable to describe the parameters of the OVRP as deterministic, because there exists much uncertainty in data such as customer demands, traveling time and the set of customers to be visited. We call these problems non-deterministic OVRP. It is very difficult or impossible to obtain fairly accurate probability distributions of the data, let alone the accurate data, the mildly perturbation of customer demands may result in the solutions far away from the optimal solution even become infeasible (Gounaris, Wiesemann, & Floudas, 2013). Demand uncertainty is a serious problem appearing in the OVRP which leads to unsatisfied demands or (and) more extra operation cost (Sungur, Ordonez, & Dessouky, 2008). Hence, it is urgent for dispatchers to develop routing and scheduling tools to directly deal with the uncertain demand in OVRP. Note that the solution of an OVRP with uncertainty differs from that of their deterministic counterpart in several fundamental respects (Cao & Lai, 2010). However, we may use the experience of solving the

* Corresponding author at: Department of Economics and Trade, Hunan University, Changsha 410079, China.
E-mail address: ceb9491@126.com (E. Cao).

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VRP with uncertainty to address the OVRP with uncertainty.The existent literature on classical VRP with uncertainty mainly focuses on stochastic vehicle routing problems (SVRP) and fuzzy vehicle routing problems (FVRP) based on mathematic programming theory. SVRPs arise whenever some elements of a given problem are random. Common examples include stochastic demands and stochastic travel times. Sometimes, the set of customers to be visited is not known with certainty. In this case, each customer has a certain probability of being visited. Researchers have developed many models and algorithms for SVRP, two basic types of stochastic programs are chance constrained models (Birge & Louveaux, 2011) and stochastic models with recourse (Christiansen, Lysgaard, & Wøhlk, 2009). Alternatively, FVRP arise whenever some elements of a given problem involve uncertainty, subjectivity, ambiguity and vagueness (Cao & Lai, 2010; Zheng & Liu, 2006). Zheng and Liu (2006) researched VRP with fuzzy travel times and presented a chance-constrained program (CCP) model with a credibility measure; they then integrated fuzzy simulation and a genetic algorithm (GA) to design a hybrid intelligent algorithm to solve the model. Cao and Lai (2010) adopted differential evolution algorithm to solve open vehicle routing problem with fuzzy demand.

Generally speaking, current stochastic programming models the uncertain parameters (for example, demand and (or) time) of VRP as random variables with known probability distribution (Birge & Louveaux, 2011). The resulting solution is potentially sensitive to the actual data that occur in the practical problem (Gounaris et al., 2013). The feasibility of the solution obtained may not be guaranteed unless the uncertainty is incorporated directly in the optimization methodology (Sungur et al., 2008). The robust optimization instead of knowing the probability distribution, only considering an uncertainty bounded set. In robust optimization, uncertainty is modeled as a set of continuous or discrete values, referred as scenarios. Thus, the general idea is to perform the optimization over a set of scenarios and to provide robust solutions over all scenarios. That is to say, the robust optimization deals directly with the robustness of the solution by finding a solution which is immune to variations in the data (Sungur et al., 2008). There is few literature considered robust vehicle routing problems. Sungur et al. (2008) first introduced the robust capacitated vehicle routing problem (RCVRP) adopted a branch-and-cut-based to solve the RCVRP under customer demands and travel times are uncertain. Lee, Lee, and Park (2012) presented a dynamic programming algorithm to solve robust vehicle routing problem with deadlines and travel time/demand uncertainty. Recently, Gounaris et al. (2013) studied the formulation and solution of the RCVRP under demand uncertainty. Agra, Christiansen, Figueiredo, Hvattum, Poss, & Requejo (2013) studied the vehicle routing problem with time windows and travel times are uncertain (modeled as an interval data), they presented two new formulations for the problem and used different robust optimization tools to handle the uncertainty.

To the best of our knowledge, no research has so far investigated the robust open vehicle routing problem (ROVRP) although the OVRP has attracted much attention with the rise of the third party logistic operation and demand uncertainty is very common in OVRP. In this paper, the proposed ROVRP is an extension of the OVRP by taking into account the uncertain demand at each customer. It is the first time to consider the robust open vehicle routing problem with uncertain demand. The main purpose of our work is to propose the effective robust strategies and investigate the effects of uncertain demand on transportation costs and unmet demands. Our work differs from the prior literature as follows: First, we consider a different problem setting: the robust open vehicle routing problem with uncertain demand, in which customers' demands belong to specific bounded uncertainty sets with expected demand value and nominal value, this problem never addressed in existing literature. Second, we propose the robust optimization model that aims at minimizing transportation costs and unsatisfied demands in the specific bounded uncertainty sets, the objectives are more closely to practical situation than only considering the cost minimum. Third, we focus on presenting the four different robust strategies and investigating the impact of uncertain demand on different robust strategies. Moreover, we adopt an improved differential algorithm to solve the model and compared the performances of four different robust strategies.

The contribution of this study is threefold. Firstly, the ROVRP model that aim at minimizing transportation costs and unsatisfied demands in the specific bounded uncertainty sets was built and the four different robust strategies to cope with the uncertain demand were proposed. Secondly, an improved differential evolution algorithm was designed to solve the ROVRP. Thirdly, the impacts of the uncertain demand on cost performance and demand performance under different robust strategies were analyzed.

The remainder of the paper is organized as follows. We describe the robust open vehicle routing problem with uncertain demand and present the robust optimization model in Section 2. We propose four robust strategies to cope with the demand uncertainty in Section 3. We present an improved differential evolution algorithm to solve the ROVRP in Section 4. We present some numeric examples and offer some qualitative discussion of solutions in Section 5. Finally, we summarize the work presented in this paper in Section 6.

2. Model OVRP with uncertain demand

2.1. Problem description

OVRP is a combinational optimization problem that intended to serve a number of customers with a fleet of vehicles, in which the vehicles are not obligated to return to the depot (Cao & Lai, 2010; Lin et al., 2013). In a typical OVRP, there is a depot (denoted 0) has $\bar{k}$ homogenous vehicles with the capacity of C units. Either each route is a sequence of customers that starts at the depot and finishes at one of the customers to whom goods are delivered, or each route is a sequence of customers that begins at a certain customer and ends at the distribution depot, where goods are gathered. Obviously, each route is open, that is to say, the vehicles do not necessarily return to their original locations after delivering goods to customers; if they do, they must visit the same customers in the reverse order. Each customer is visited exactly once by one vehicle, while vehicle activity is bounded by capacity constraints. The load is u_{ik} for vehicle k after visiting customer i, and the distances is c_{ij} between customer i and customer j. We assume the actual demand is unknown at the time when the tour is designed, but is known before the vehicles start its route. The objectives are to minimize total transportation cost and unmet demand for a fleet of vehicles that serve some commodity to a given number of customers. Similar to Sungur et al. (2008), we assume that the demand d_i of customer i is uncertain and belongs to a bounded set $U_D = \left\{d|d^0 + \sum_{t=1}^{T} y_t d^t, y \in Y\right\}$, where uncertainty sets including an expected demand value (or nominal demand values d^0)and a possible deviation directions from these expected values are fixed and identified by scenario vectors $d^t \in R^n$. We assume there are T scenarios and the demand is decreasing when the scenario vectors have negative deviation values. In particular, we consider the following three set for U_D. Convex hull $U_{D1} = \left\{d|d^0 + \sum_{t=1}^{T} y_t d^t, y \in R^T, y \geq 0, \sum_{t=1}^{T} y_t \leq 1\right\}$; Box $U_{D2} = \left\{d|d^0 + \sum_{t=1}^{T} y_t d^t, y \in R^T, \|y\|_\infty \leq 1\right\}$; Ellipsoidal $U_{D3} = \left\{d|d^0 + \sum_{t=1}^{T} y_t d^t, y \in R^T, y^T Q y \leq 1\right\}$, Q is a positive definite matrix.