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Review

A parameter selection strategy for particle swarm optimization based on particle positions

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ABSTRACT

In this study, we found that engineering experience can be used to determine the parameters of an optimization algorithm. We came to this conclusion by analyzing the dynamic characteristics of PSO through a large number of experiments. We constructed a relationship between the dynamic process of particle swarm optimization and the transition process of a control system. A novel parameter strategy for PSO was proven in this paper using the overshoot and the peak time of a transition process. This strategy not only provides a series of flexible parameters for PSO but it also provides a new way to analyze particle trajectories that incorporates engineering practices. In order to validate the new strategy, we compared it with published results from three previous reports, which are consistent or approximately consistent with our new strategy, using a suite of well-known benchmark optimization functions. The experimental results show that the proposed strategy is effective and easy to implement. Moreover, the new strategy was applied to equally spaced linear array synthesis examples and compared with other optimization methods. Experimental results show that it performed well in pattern synthesis.

1. Introduction

Particle swarm optimization (PSO), a stochastic optimization method, is an evolutionary simulation algorithm derived from the behavior of flocks of birds and schools of fish. Animals, especially birds and fish, travel in groups without ever colliding; each member follows the group and adjusts its position and velocity using information from the group. PSO, modeled after this swarming behavior, is a powerful tool for optimizing a wide range of problems. To gain deeper insight into the mechanism of PSO, many theoretical analyses have been conducted on the algorithm using deterministic or stochastic methods. Clerc and Kennedy simplified PSO to a deterministic dynamical system and mathematically analyzed the stochastic behavior of the PSO algorithm (Clerc & Kennedy, 2002). Such simplified, deterministic versions of PSO or similar systems, which employ a single particle, fixed attractors, or constant coefficients, have been analyzed by many researchers for stability, convergence, and parameter selection. Accelerating the convergence speed and avoiding the local optimal solution are two main goals that drive the study of PSO. There are many factors that affect the convergence and performance of the PSO algorithm, such as swarm size, velocity clamping, position clamping, topology of neighborhoods, and synchronous or asynchronous updates. In addition, values of the inertia weight and the acceleration coefficients (the cognitive part and the social part of the model, respectively) may significantly affect the efficiency and reliability of the PSO. Proper selection of these two parameters can improve the convergence rate of PSO. Furthermore, some theoretical analyses of particle trajectories have provided insights into how the particle swarm system works. Trelea analyzed the dynamic behavior and the convergence of a simplified PSO algorithm using standard results from the discrete-time dynamical system theory and provided a parameter set in the algorithm's convergence domain (Trelea, 2003). Eberhart and Shi empirically found that an inertia weight of 0.729 and acceleration coefficients of 1.496 were good parameter choices that led to convergent trajectories (Eberhart & Shi, 2000). Similarly, Jiang et al. studied the stochastic convergence property of the standard PSO algorithm and reported on a sufficient condition that ensured the stochastic convergence of the particle swarm system (Jiang, Luo, & Yang, 2007b). Subsequently, according to these analysis results, a set of suggested PSO parameters was provided in another report (Jiang, Luo, & Yang, 2007a).

Researchers have also attempted various ways to analyze and improve PSO. Kennedy proposed a PSO where the usual velocity formula was removed and replaced by samples from a Gaussian distribution (Kennedy, 2003). Although the method greatly simplified the particle swarm algorithm, the performance of the Gaussian version of PSO was worse than the canonical PSO (Kennedy, 2004). Sun et al. proposed quantum-behaved particle swarm optimization (QPSO), motivated by concepts in quantum mechanics, to improve

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the global search ability of PSO (Sun et al., 2012; Xi, Sun, & Xu, 2008). They first employed the theory of probabilistic metric spaces to analyze the convergence of QPSO. Then, Zhou et al. introduced a random position PSO to improve the global search ability of particle swarm optimization. Similarly Zhou et al. (2011), and Chen et al. developed a novel hybrid algorithm that combined PSO with extremal optimization (EO) to avoid premature convergence of PSO (Chen, Li, Zhang, & Lu, 2010). Moreover, similar variants are continually being devised using the concept of disturbance (Zhao, 2010), fuzzy logic (Melin et al., 2013), and hybrid methods that involve various evolutionary algorithms, such as ant colony optimization (ACN) (Niknam & Amiri, 2010), differential evolution (DE) (Liu, Cai, & Wang, 2010), artificial neural networks (ANNs) (Vasumathi & Moorthi, 2012), and chaos optimization algorithms (COA) (Jiang, Kwong, Chen, & Ysim, 2012).

Most of the above algorithms improve PSO performance in some ways, but they also increase the algorithmic complexity. Pedersen et al. have demonstrated that satisfactory performance can be achieved with basic PSO when its parameters are tuned properly (Pedersen & Chipperfield, 2010). Martínez et al. proposed some promising parameter sets, which resulted in a good parameter region of the inertia value and acceleration coefficients (Martínez & Gonzalo, 2008). In order to capture the stochastic behavior of the entire swarm, Chen and Jiang analyzed the particle interactions and proposed a statistical interpretation of PSO (Chen & Jiang, 2010). In a word, the above reports provide insights, based on mathematical analyses, into how the particle swarm system works. Although PSO is now widely applied in many research fields, the theoretical analysis of PSO is still quite limited. Furthermore, oscillation properties also have an important influence on the optimization process, and, so far, there have been few theoretical analyses on optimization process based on control theory. The "No Free Lunch" theorem (Wolpert & Macready, 1997) and "Optimal Contraction Theorem" (Chen, Xin, Peng, Dou, & Zhang, 2009) indicate that no optimizers can be optimal for arbitrary problems, and a balance between exploitation and exploration in PSO is desirable for any problem. To enhance the searching ability of PSO and accelerate its convergence, we performed a detailed theoretical and empirical analysis, and we propose a parameter selection scheme in this paper.

The remainder of this paper is organized as follows: The new parameter selection strategy for PSO, which was developed according to our analysis of particle positions, is provided in Section 2. A comparative analysis of the new parameter selection strategy, along with the parameter selection rule raised in the literature (Jiang et al., 2007a) is presented in Section 3. Experiments on numerical optimization used to illustrate the efficiency of the proposed parameter selection strategy are given in Section 4. Section 5 gives the application of the new strategy to antenna array pattern synthesis. Finally, a conclusion and future research are given in Section 6.

2. Characteristic analysis of standard particle swarm optimization and introduction to the new parameter selection strategy

Standard particle swarm optimization (SPSO) maintains a swarm of particles representing candidate solutions for a given optimization problem. The movement dynamics of each particle in the search space is governed by its current position and velocity which can be regarded as the potential solution in the D-dimension problem space, whereby the current velocity is determined by (1) its previous velocity, (2) its distance to the position where the particle achieved its best fitness (personal best, p), and (3) its distance from the particle that achieved the best fitness among all the particles (global best, g). The position of each particle is a potential solution, and each particle memorizes the best position it achieves during the entire optimization process (p). The swarm as a whole memorizes the best position achieved by any of its particles (g). The position and the velocity relationship after the kth iteration between any two individuals is obtained by the following updating formula:

$$v[k+1] = \omega \cdot v[k] + c_p \cdot r_p[k] \cdot (p[k] - x[k]) + c_g \cdot r_g[k] \cdot (g[k] - x[k]) \tag{1}$$

$$x[k+1] = x[k] + v[k+1] \tag{2}$$

where ω is a parameter called the inertia weight, and the acceleration factors c_p and c_g are positive constants that control the relative impact of the personal (local) and common (global) knowledge on the movement of each particle. The terms r_p and r_g are independent, uniformly distributed random variables in the range of (0, 1), and $p[k]$ is the best previous position of $x[k]$ while $g[k]$ is the best overall position achieved by a particle within the entire population. In this version, $v[k]$ is clamped to a maximum magnitude $V_{\max}$.

In the PSO algorithm, proper control of global exploration and local exploitation is a crucial issue. Shi and Eberhart introduced the concept of inertia weight to the original version of PSO to balance the local and global searches during the evolution process (Shi & Eberhart, 1998). In general, a large inertia weight imposed at the early search stages allows the search space to be thoroughly explored. By gradually decreasing the inertia weight, more refined solutions are achieved in the final search stage. The major goal of this modification was to avoid premature convergence in the early search stages and to improve convergence to the global optimal solution during the latter search stages. The concept of linearly decreasing inertia weight applied to particle swarm optimization (LPSO) was introduced in (Shi & Eberhart, 1999) and is given by:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} iter, \tag{3}$$

where $\omega_{\max}$ and $\omega_{\min}$ are the initial and final values of the inertia weight, respectively, $iter$ is the current iteration number, and $iter_{max}$ is the maximum number of allowable iterations. Usually, parameters $\omega_{\max}$ and $\omega_{\min}$ are set to 0.9 and 0.4, respectively. Therefore, the particles use a larger inertia weight during the initial exploration and use lower inertia weight values as the search progresses to later iterations.

Analyzing the behavior of particles by studying particle trajectories and identifying factors that influence the dynamic behavior characteristics of particles is necessary.

By substituting Eq. (1) into Eq. (2), we obtain the following nonhomogeneous recurrence relation:

$$\begin{aligned} x[k+1] + (c_p \cdot r_p[k] + c_g \cdot r_g[k] - 1 - \omega) \cdot x[k] + \omega \cdot x[k-1] \\ = c_p \cdot r_p[k] \cdot p[k] + c_g \cdot r_g[k] \cdot g[k] \end{aligned} \tag{4}$$

Applying the expectation operator to both sides of Eq. (4), we obtain Eq. (5):

$$\begin{aligned} Ex[k+2] + \left(\frac{c_p + c_g}{2} - 1 - \omega\right) \cdot Ex[k+1] + \omega \cdot Ex[k] \\ = \frac{c_p \cdot p[k] + c_g \cdot g[k]}{2} \end{aligned} \tag{5}$$

The convergence analysis regarding the expectation of a particle's position was proven by Jiang et al. (2007b) in detail. They introduced the following theorem:

Given ω, c_p, $c_g \geqslant 0$, if and only if $0 \leqslant \omega < 1$ and $0 < c_p + c_g < 4(1 + \omega)$, iterative process $\{EX[k]\}$is guaranteed to converge to $(c_p.p + c_g.g)/(c_p + c_g)$.