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Fuzzy linear regression based on Polynomial Neural Networks

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ABSTRACT

In this study, we introduce an estimation approach to determine the parameters of the fuzzy linear regression model. The analytical solution to estimate the values of the parameters has been studied. The issue of negative spreads of fuzzy linear regression makes the problem to be NP complete. To deal with this problem, an iterative refinement of the model parameters based on the gradient decent optimization has been introduced.

In the proposed approach, we use a hierarchical structure which is composed of dynamically accumulated simple nodes based on Polynomial Neural Networks the structure of which is very flexible.

In this study, we proposed a new methodology of fuzzy linear regression based on the design method of Polynomial Neural Networks. Polynomial Neural Networks divide the complicated analytical approach to estimate the parameters of fuzzy linear regression into several simple analytic approaches.

The fuzzy linear regression is implemented by Polynomial Neural Networks with fuzzy numbers which are formed by exploiting clustering and Particle Swarm Optimization. It is shown that the design strategy produces a model exhibiting sound performance.

1. Introduction

In recent years, the problem of modeling and prediction from observed data has been one of the most commonly encountered research topics in machine learning and data analysis (Guvenir & Uysal, 2000).

A simple way to describe a system is the regression analysis (Yu & Lee, 2010). In the classical regression both independent and dependent variables are treated as real numbers. However, in many real-world situations, where the complexity of the physical system calls for the development of a more general viewpoint, regression variables are specified in the form of some non-numeric (granular) entities such as e.g., linguistic variables (Cheng & Lee, 2001). The well-known and commonly encountered classical regression cannot handle such situations (Bardossy, 1990; Bardossy, Bogardi, & Duckstein, 1990).

The fuzzy regression, which can deal with the non-numerical entities, especially linguistic variables, was proposed by Imoto, Yabuuchi, and Watada (2008), Tanaka, Uejima, and Asai (1982), Toyoura, Watada, Khalid, and Yusof (2004), and Watada (2001). A fuzzy linear regression proposed by Tanaka is composed of the numeric input variables and the linguistic (granular) coefficients which are treated as some fuzzy numbers (in particular, those are described by triangular membership functions). The linguistic coefficients of the regression lead to the linguistic output of the regression model. In other words, the output of a fuzzy linear regression model becomes also a triangular fuzzy number.

In essence, the fuzziness of the output of the regression model emerged because of the lack of perfect fit of numeric data to the assumed linear format of the relationship under consideration. In other words, through the introduction of triangular numbers (parameters of the model), this fuzzy regression reflects the deviations between the data and the linear model. Computationally, the estimation of the fuzzy parameters of the regression is concerned with some problems of linear programming (Bargiela, Pedrycz, & Nakashima, 2007).

Diamond developed a simple regression model for triangular fuzzy numbers under the conceptual framework as

$$F(\mathbf{R}^m) \rightarrow F(\mathbf{R}) \tag{1}$$

where $F(\mathbf{R})$ denotes a family of fuzzy numbers (in our case triangular ones) defined in the space of real numbers $\mathbf{R}$.

For the conceptual framework formed by (1), the various analytical formulae quantifying the values of the parameters of the regression model had to address the issue of negative spreads (Diamond & Koerner, 1997), which complicates significantly the algorithms and makes them difficult to apply to highly-dimensional data.

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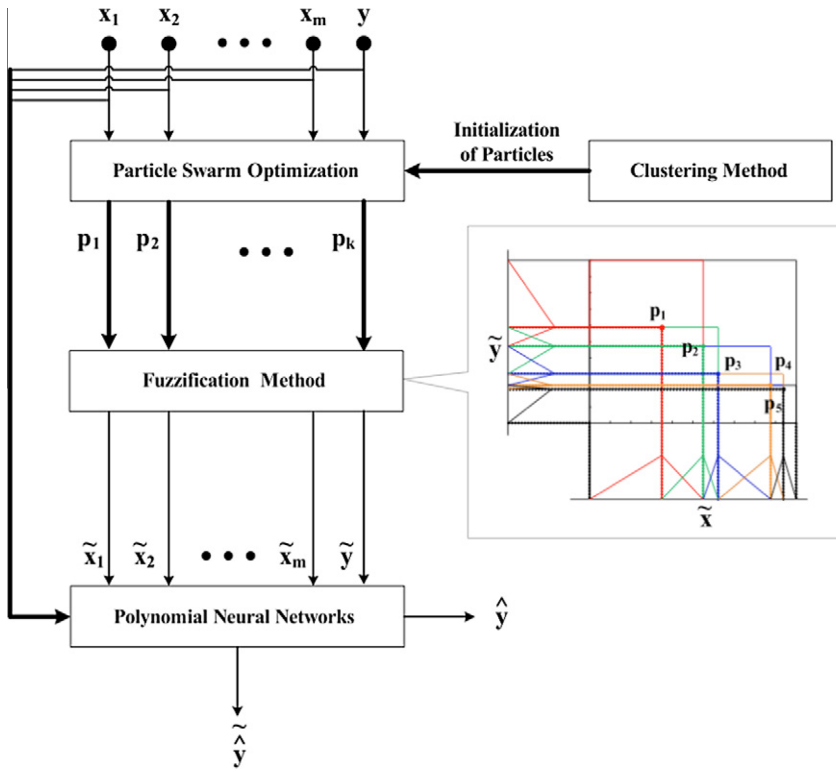

Fig. 1. An overall structure of the Polynomial Neural Network with fuzzy data.

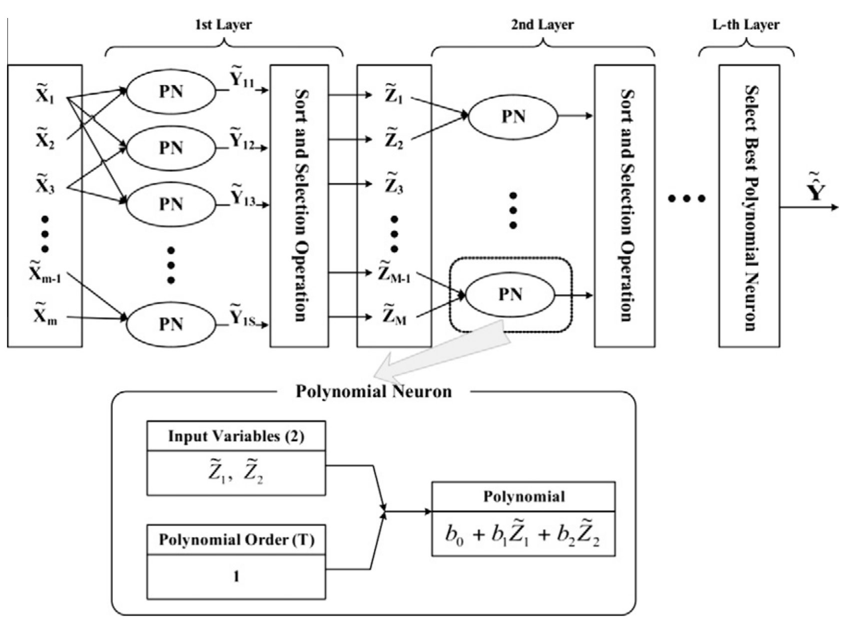

Fig. 2. An overall structure of the PNN.

Considering the optimization standpoint, A. Bargiela et al. (2007) revised the mapping between the independent variables and the dependent variable to be expressed as follows

$$F(\mathbf{R}) \times F(\mathbf{R}) \times \cdots \times F(\mathbf{R}) \rightarrow F(\mathbf{R}) \tag{2}$$

In addition, to deal with the issue of negative spreads, A. Bargiela proposed a certain re-formulation of the regression problem as a gradient-descent optimization task, which enables a generic generalization of the simple regression model to multiple regression models in a computationally feasible way (Toyoura et al., 2004).

The iterative refinement based on the gradient decent approach to estimate the parameter of fuzzy linear regression is the modification of the conventional gradient decent optimization. The drawback of the gradient decent optimization is well-known: the optimization performance of the gradient decent optimization