Expert Systems with Applications 39 (2012) 2578–2583

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Combining an alternative multi-delivery policy into economic production lot size problem with partial rework

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ARTICLE INFO

Keywords:
Replenishment lot size
Discontinuous issuing policy
EPQ model
Multiple deliveries
Rework
Scrap

ABSTRACT

This paper combines an alternative multi-delivery policy into an imperfect economic production quantity (EPQ) model with partial rework, with the purpose of reducing supplier's stock holding cost. We extend the problem examined by Chiu et al. [Chiu, Y.-S. P, Chiu, S. W., Li, C.-Y., & Ting, C.-K. (2009). Incorporating multi–delivery policy and quality assurance into economic production lot size problem. *Journal of Scientific and Industrial Research*, *68*(6), 505–512] by considering an $n+1$ delivery policy in lieu of n multi-delivery plan for the specific EPQ model with partial rework. Under such a policy, an initial delivery of perfect (finished) items is distributed to customer for satisfying product demand during manufacturer's regular production time and rework time. At the end of rework, fixed quantity n installments of the finished products are delivered to customer at a fixed interval of time. As a result, a closed-form optimal replenishment batch size solution to the problem is obtained. A numerical example with analysis and comparison is provided to show practical usage of the proposed model and demonstrate its significant savings in stock holding cost.

1. Introduction

The economic production quantity (EPQ) model makes use of mathematical techniques to balance the fixed production setup cost and stock holding cost during production process, and to derive the optimal replenishment lot size that assists manufacturing firms in minimizing the long-run average cost per unit time (Hadley & Whitin, 1963; Hillier & Lieberman, 2001; Nahmias, 2009; Silver, Pyke, & Peterson, 1998). The classic EPQ model implicitly assumes that all items produced are of perfect quality. However, in real-life production systems, due to process deterioration or other controllable and/or uncontrollable factors, generation of defective items is inevitable. Hence, many studies have been carried out to enhance the EPQ model by addressing the imperfect quality issues (Henig & Gerchak, 1990; Makis, 1998; Rosenblatt & Lee, 1986; Shih, 1980; Wee, Yu, & Chen, 2007). Imperfect quality items sometimes can be reworked and repaired, and hence overall production costs can be significantly reduced (Agnihothri & Kenett, 1995; Chen, Liao, & Weng, 2007; Chiu & Chiu, 2006; Chiu, Chen, Cheng, & Wu, 2010; Grosfeld-Nir & Gerchak, 2002; Yum & McDowell, 1987). For instance, manufacturing processes in printed circuit board assembly, or in plastic injection molding, etc., sometimes employs rework as an acceptable process in terms of level of quality. Grosfeld-Nir and Gerchak (2002) considered multistage production systems where defective units can be reworked repeatedly at every stage. They showed that a multistage system where only one of the stages requires a set-up can be reduced to a single-stage system. They proved that it is best to make the "bottle-neck" the first stage of the system and they also developed recursive algorithms for solving two- and three-stage systems. Chiu and Chiu (2006) proposed a mathematical modeling for determining optimal batch size and backordering level for a finite production rate model with backlogging and failure in repair.

Continuous inventory issuing policy for satisfying product demand is another unrealistic assumption of the classic EPQ model. In real-life vendor–buyer integrated production–inventory system, multiple or periodic deliveries of finished products are commonly adapted. Schwarz (1973) examined a one-warehouse N-retailer deterministic inventory system. The objective was to determine the stocking policy which minimizes average system cost per unit time. Necessary properties of an optimal policy were derived and the optimal solutions for the one-retailer and N identical retailer problems were given. Heuristic solutions for the general problem were also suggested, tested against analytical lower bounds and on the basis of these tests, found to be near optimal. Goyal (1977) studied the integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. An example

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doi:10.1016/j.eswa.2011.08.112

was provided to illustrate his proposed method. Many studies have since been carried out to address various aspects of supply chain optimization (Abdul-Jalbar, Gutiérrez, & Sicilia, 2005; Banerjee, 1986; Bilgen, 2010; Buscher & Lindner, 2005; Chiu, Chiu, Li, & Ting, 2009; Diponegoro & Sarker, 2006; Goyal & Gupta, 1989; Goyal & Nebebe, 2000; Hill, 1996; Kim, Banerjee, & Burton, 2008; Ojha, Sarker, & Biswas, 2007; Sarker & Parija, 1994; Sarker & Khan, 2001; Su & Wong, 2008; Viswanathan, 1998). Examples of studies are surveyed as follows.

Sarker and Parija (1994) considered a manufacturing system which procures raw materials from suppliers and processes them to convert to finished products. They proposed a model that was used to determine an optimal ordering policy for procurement of raw materials, and the manufacturing batch size to minimize the total cost for meeting equal shipments of the finished products, at fixed intervals, to the buyers. Viswanathan (1998) reexamined the integrated vendor–buyer inventory models with two different strategies that had been proposed in the literature for the problem: one where each replenishing quantity delivered to the buyer is identical and the other strategy where at each delivery all the inventory available with the vendor is supplied to the buyer. He showed that there is no one strategy that obtains the best solution for all possible problem parameters. Goyal and Nebebe (2000) considered the problem of determining economic production and shipment policy of a product supplied by a vendor to a single buyer. The objective of their study was to minimize the total joint annual costs incurred by the vendor and the buyer. Buscher and Lindner (2005) presented a lot size model which addresses a production system with rework and equal sized batch shipments. Based on total relevant costs per unit time, an optimization method was developed to determine the economic production and rework quantity as well as the corresponding batch sizes for both activities. The algorithm was illustrated by a numerical example followed by a sensitivity analysis of the models behavior under different problem parameters. Diponegoro and Sarker (2006) determined an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was then extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. A lower bound on the optimal solution was also developed for problem with lost sale. It was shown that the solution and the lower bound were consistently tight. Chiu et al. (2009) incorporated a multi-delivery policy and quality assurance into an imperfect economic production quantity (EPQ) model with scrap and rework. They assumed that the random defective items produced are partially repairable and are reworked in each cycle when regular production ends, and the finished items can only be delivered to customers if the whole lot is quality assured at the end of rework. Fixed quantity multiple installments of the finished batch are delivered to customers at a fixed interval of time. The expected integrated cost function per unit time was derived. A closed-form optimal batch size solution to the problem was obtained. This paper reexamines the problem studied by Chiu et al. (2009), with the purpose of reducing supplier's stock holding cost, an $n + 1$ delivery policy is considered in lieu of n multi-delivery plan for this specific EPQ model with partial rework. The joint effects of the $n + 1$ multi-delivery policy and partial rework on the optimal replenishment lot size of the imperfect EPQ model are investigated.

2. Description and modeling

Chiu et al. (2009) studied an extended EPQ model which incorporated quality assurance issue and a multi-delivery policy into the classic EPQ model. The quality assurance issue is in regard to the production system which has an x portion of random defective items produced at a production rate d, and among defective items, a θ portion is assumed to be scrap and the other $(1 - \theta)$ portion can be reworked and repaired at a rate P_1, within the same cycle when regular production ends. While the multi-delivery policy is with regard to fixed quantity of n installments of the finished batch are delivered to customer at a fixed interval of time during the production downtime (i.e. when the whole lot is quality assured at the end of rework).

With the purpose of reducing supplier's stock holding cost, this paper reexamines the lot-size problem studied by Chiu et al. (2009), an $n + 1$ delivery policy is considered here in lieu of n multi-delivery plan for this specific EPQ model with partial rework. Fig. 1 depicts the on-hand inventory of perfect quality items of the proposed model. Fig. 2 illustrates the expected reduction in inventory holding costs (in yellow/shade areas) of the proposed model (in blue) in comparison with the model studied by Chiu et al. (2009) (in red).

Assumption for regular supply in EPQ model remains, that is constant production rate P has to be larger than sum of demand rate λ and production rate d. Thus, $(P - d - \lambda) > 0$ or $(1 - x - \lambda/P) > 0$; where $d = Px$. Cost parameters considered in the proposed model include unit production cost$C$, unit holding cost$h$, setup cost$K$ per production run, unit rework cost C_R, disposal cost per scrap item C_S, holding cost h_1 for each reworked item, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Additional notation is listed as follows.

Q	production lot size to be determined for each cycle
H	the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time t_1 and rework time t_2
H_1	maximum level of on-hand inventory in units when regular production ends
H_2	the maximum level of on-hand inventory in units when rework process finishes
T	cycle length
t	the production time needed for producing enough perfect items for satisfying product demand during production time t_1 and rework time t_2
t_1	the production uptime for the proposed EPQ model
t_2	time required for reworking of defective items
t_3	time required for delivering the remaining quality assured finished products
n	number of fixed quantity installments of the finished batch to be delivered to customer during t_3
t_n	a fixed interval of time between each installment of products delivered during t_3
$I(t)$	on-hand inventory of perfect quality items at time t

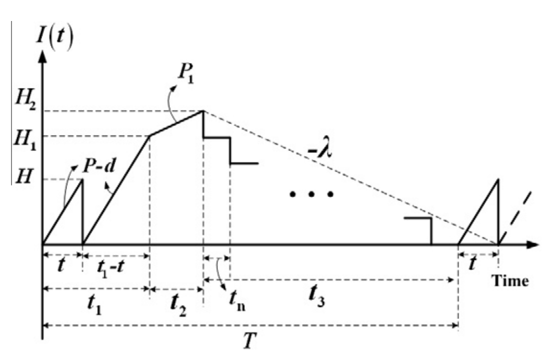

Fig. 1. On-hand inventory of perfect quality items in EPQ model with $(n+1)$ delivery policy and partial rework.