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New aggregation operators based on the Choquet integral and 2-tuple linguistic information ☆

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ABSTRACT

Considered in this paper is the group decision making problem with inter-dependent or interactive attributes, where evaluation values of decision makers are in linguistic arguments. By using the Choquet integral, some new aggregation operators are introduced, including the 2-tuple correlated averaging operator, the 2-tuple correlated geometric operator and the generalized 2-tuple correlated averaging operator. The proposed operators can better reflect the correlations among the elements. After investigating properties of these operators, a new multiple attribute decision making method based on the new operators is proposed. Finally, a numerical example is provided to illustrate the feasibility and efficiency of the proposed method.

1. Introduction

The 2-tuple fuzzy linguistic representation model was developed by Herrera and Martinez (2000a) on the basis of the concept of symbolic translation. It can avoid the information distortion and lose in the linguistic information processing. The 2-tuple linguistic model has been examined and applied in the decision making (Chang & Wen, 2010; Dong, Xu, & Yu, 2009a, Dong, Xu, & Yu, 2009b; Dong, Xu, Li, & Feng, 2010; Dursun & Karsak, 2010; Herrera & Martinez, 2000a; Herrera, Herrera-Viedma, & Martinez, 2000; Herrera-Viedma, Martinez, Mata, & Chiclana, 2005; Herrera & Martinez, 2000b; Herrera & Martinez, 2001; Herrera-Viedma, Mata, Martinez, & Chiclana, 2005; Jiang & Fan, 2003; Martinez, 2007; Moreno, Morales Des Castillo, Porcel, & Herrera-Viedma, 2005; Wang, 2008; Wang, 2009; Wang, 2010; Wang & Klir, 1992; Wang & Fan, 2003; Wang & Hao, 2006; Xu, 2004; Wei, 2008, 2010a, 2010b; Wei, Lin, Zhao, & Wang, 2010; Zhang & Fan, 2006). Many aggregation operators have been proposed. Herrera and Martinez (2000b) defined the 2-tuple arithmetic averaging operator, the 2-tuple arithmetic weighted averaging operator, the 2-tuple ordered weighted averaging operator and the extended 2-tuple weighted averaging operator. Xu (2004) introduced the extended geometric mean operator, the extended arithmetic averaging operator, the extended ordered weighted averaging operator and the extended ordered weighted geometric operator. Jiang and Fan (2003) developed the 2-tuple ordered weighted averaging operator and the 2-tuple ordered weighted geometric operator. Zhang and Fan (2006) presented the extended 2-tuple ordered weighted averaging operator. The extended 2-tuple weighted geometric operator and the extended 2-tuple ordered weighted geometric operator have been proposed by Wei (2010b).

In the above aggregation operators, the attributes are assumed to be independent of one another, which are characterized by an independent axiom Wakker (1999). But in the real decision making process, the attributes of the problem are often inter-dependent or correlated. Choquet integral was introduced by Choquet (1953) and is a useful tool to model the inter-dependence or correlation. It has been studied and applied in the decision making (Angilella, Greco, Lamantia, & Matarazzo, 2004; Angilella, Greco, & Matarazzo, 2010; Büyüközkan & Ruan, 2010; Demirel, Demirel, & Kahraman, 2010; Grabisch & Labreuche, 2010; Hu, 2000; Meyer & Roubens, 2006; Labreuche & Grabisch, 2003; Labreuche & Grabisch, 2006; Saad, Hammadi, Benrejeb, & Borne, 2008; Shieh, Wu, & Liu, 2009; Tan & Chen, 2010a, 2010b; Tseng, Yang, Lin, & Chen, 2005; Tseng, Chiang, & Lan, 2009; Xu, 2009; Xu, 2010; Yager, 2003, 2009). Yager (2003) defined the induced Choquet ordered averaging operator to aggregate a collective real arguments. The intuitionistic fuzzy sets has been aggregated with Choquet integral in Yager (2009). The intuitionistic fuzzy Choquet integral operator was developed by Tan and Chen (2010a) and the induced Choquet ordered averaging operator was presented in Tan and Chen (2010b). Xu (2010) proposed the intuitionistic fuzzy correlated averaging operator, the intuitionistic fuzzy correlated geometric operator, the interval-valued intuitionistic fuzzy correlated averaging operator and the

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interval-valued intuitionistic fuzzy correlated geometric operator to aggregate the intuitionistic fuzzy information or the interval-valued intuitionistic fuzzy information. Meyer and Roubens (2006) aggregated the fuzzy numbers through the Choquet integral and further presented a fuzzy extension of the Choquet integral. Until now, we have not found any aggregation of the 2-tuple linguistic information with the Choquet integral. Some new Choquet integral-based aggregation operators with the 2-tuple linguistic information are developed in this paper.

The rest of the paper is organized as follows: some basic concepts are illustrated in Section 2. In Section 3, several new aggregation operators are introduced, concretely, the 2-tuple correlated averaging (TCA) operator, the 2-tuple correlated geometric (TCG) operator and the generalized 2-tuple correlated averaging (GTCA) operator. Some special cases of the proposed operators are examined. The properties of these operators are studied. The multiple attribute decision making method based on these new operators is then proposed in Section 4. In Section 5, a numerical example is given to illustrate the developed approach and to demonstrate its feasibility and practicality. In the last section, we give the conclusions.

2. Basic concepts

As a preparation for introducing our new aggregation operators, some relevant concepts are illustrated in this section.

Suppose that $S = \{s_i | i = 0, \ldots, t\}$ is a finite and totally ordered discrete term set, where s_i represents a possible linguistic term for a linguistic variable. For example, a set of seven terms can be expressed as

$$S = \{s_1 = \textit{extremely poor}\ (\mathrm{EP}), s_2 = \textit{very poor}\ (\mathrm{VP}), s_3 = \textit{poor}\ (\mathrm{P}), s_4 = \textit{medium}\ (\mathrm{M}), s_5 = \textit{good}\ (\mathrm{G}), s_6 = \textit{very good}\ (\mathrm{VG}), s_7 = \textit{extremely good}(\mathrm{EG})\}.$$

The above set satisfies the following properties:

(1) The set is ordered: $s_i \geqslant s_j$, if $i \geqslant j$;
(2) The max operator: $\max(s_i, s_j) = s_i$, if $i \geqslant j$;
(3) The min operator: $\min(s_i, s_j) = s_i$, if $i \leqslant j$.

The 2-tuple fuzzy linguistic representation model has been developed by Herrera and Martinez (2000a) basing on the concept of symbolic translation. The 2-tuple (s_i, α_i) is used to represent the linguistic information, where s_i is a linguistic label from a predefined linguistic term set S and α_i is a numerical value representing the value of the symbolic translation and $\alpha_i \in [-0.5, 0.5)$

Definition 1 Herrera and Martinez (2000a). Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation, $\beta \in [1, t]$, with t being the cardinality of S. Let $i = round(\beta)$, here $round(\cdot)$ is the usual round operation, and $\alpha = \beta - i$ such that $i \in [1, t]$ and $\alpha \in [-0.5, 0.5)$, then α is called a symbolic translation.

Definition 2 Herrera and Martinez (2000a). Let S be a linguistic term set and β be a number representing the aggregation result of linguistic symbolic. The function Δ used to obtain the 2-tuple linguistic information equivalent to β is defined as:

$$\Delta : [1, t] \to S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = \begin{cases} s_i, & i = round(\beta), \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{2}$$

where s_i has the closest index label to β and α is the value of the symbolic translation.

Definition 3 Herrera and Martinez (2000a). Let S be a linguistic term set, (s_i, α_i) be a 2-tuple. The function Δ^{-1} from a 2-tuple (s_i, α_i) to its equivalent numerical value $\beta \in [1, t] \subset R$ can be defined as follows:

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [1, t], \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta, \tag{4}$$

Definition 4 Herrera and Martinez (2000a). Let (s_i, a_i) and (s_j, a_j) be two 2-tuples, they have the following properties:

(1) If $i < j$, then (s_i, a_i) is smaller than (s_j, a_j),
(2) If $i = j$, and
  (a) if $a_i = a_j$, then (s_i, a_i) and (s_j, a_j) represent the same information;
  (b) if $a_i < a_j$, then (s_i, a_i) is smaller than (s_j, a_j);
  (c) if $a_i > a_j$, then (s_i, a_i) is bigger than (s_j, a_j).

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the set of the attributes, $P(X)$ be the power set of X and $\mu(x_i)$ $(i = 1, 2, \ldots, n)$ be the weights of the elements $x_i \in X (i = 1, 2, \ldots, n)$, where μ is a fuzzy measure, defined as follows:

Definition 5 Wang and Klir (1992). A fuzzy measure μ on the set X is a set function $\mu : P(X) \to [0, 1]$ satisfying the following axioms:

(1) $\mu(\phi) = 0$, $\mu(X) = 1$;
(2) $B \subseteq C$ implies $\mu(B) \leqslant \mu(C)$, for all $B, C \subseteq X$;
(3) $\mu(B \cup C) = \mu(B) + \mu(C) + \rho\mu(B)\mu(C)$ for all B, $C \subseteq X$ and $B \cap C = \phi$, where $\rho \in (-1, +\infty)$.

In the above definition, if $\rho = 0$, then the third condition reduces to the axiom of the additive measure:

$$\mu(B \cup C) = \mu(B) + \mu(C) \text{ for all } B, \quad C \subseteq X \text{ and } B \cap C = \phi. \tag{5}$$

If the elements of B in X are independent, we have

$$\mu(B) = \sum_{x_i \in B} \mu(x_i), \text{ for all } B \subseteq X. \tag{6}$$

3. 2-Tuple linguistic information aggregation operators based on the Choquet integral

In this section, we will introduce some new aggregation operators with correlative weights by using the Choquet integral.

Definition 6. Let (r_1, a_1), $(r_2, a_2), \ldots,$ (r_n, a_n) be 2-tuple linguistic arguments, X be the set of attributes and μ be a fuzzy measure on X, then the 2-tuple correlated averaging (TCA) operator is defined as follows:

$$\mathrm{TCA}_\mu((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) = \Delta\left(\sum_{i=1}^{n} (\mu(H_{\sigma(i)}) - \mu(H_{\sigma(i-1)}))\Delta^{-1}(r_{\sigma(i)}, a_{\sigma(i)})\right), \tag{7}$$

here $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $(r_{\sigma(1)}, a_{\sigma(1)}) \geqslant (r_{\sigma(2)}, a_{\sigma(2)}) \geqslant \cdots \geqslant (r_{\sigma(n)}, a_{\sigma(n)})$, $x_{\sigma(i)}$ is the attribute corresponding to $(r_{\sigma(i)}, a_{\sigma(i)})$. $H_{\sigma(i)} = \{x_{\sigma(k)} | k \leqslant i\}$ for $i \geqslant 1$, $H_{\sigma(0)} = \phi$.

We now consider some special cases of the TCA operator. Let (r_i, a_i) $(i = 1, 2, \ldots, n)$ be 2-tuple linguistic arguments and μ be a fuzzy measure on X.