Expert Systems with Applications 38 (2011) 5205–5211

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

An OWA-TOPSIS method for multiple criteria decision analysis

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ARTICLE INFO

Keywords:
Multiple criteria decision analysis
TOPSIS
OWA
Distance-based ranking
Decision aggregation

ABSTRACT

A hybrid approach integrating OWA (Ordered Weighted Averaging) aggregation into TOPSIS (technique for order performance by similarity to ideal solution) is proposed to tackle multiple criteria decision analysis (MCDA) problems. First, the setting of extreme points (ideal and anti-ideal points) in TOPSIS is redefined and extended for handling the multiple extreme points situation where a decision maker (DM) or multiple DMs can provide more than one pair of extreme points. Next, three different aggregation schemes are designed to integrate OWA into the TOPSIS analysis procedure. A numerical example is provided to demonstrate the proposed approach and the results are compared for different aggregation settings and confirm the robustness of rankings from different scenarios.

1. Introduction

Due to ever increasing complexity of human society, people often need to consider multiple criteria (attributes, factors, objectives) to make decisions. The research area of multiple criteria decision analysis (MCDA) is developed to provide decision aid for complex decision situations. MCDA aims to furnish a set of decision analysis techniques to help decision makers (DMs) logically identify, compare, and evaluate alternatives according to diverse, usually conflicting, criteria arising from societal, economic, and environmental considerations. This body of literature has also been interexchangeably referred to as multiple criteria (attribute) decision aid (making) (Figueira, Greco, & Ehrgott, 2005).

MCDA provides a systematic framework to investigate complex decision problems containing multiple intertwining criteria. MCDA concentrates on decision analysis with a finite set of alternatives and offers a host of methods for preference elicitation and aggregation. A unique feature of MCDA is *preference-based aggregation*. To reach a final recommendation, it is inevitable that an aggregation procedure is required to synthesize alternatives' performances over different criteria. To achieve this more effectively, the aggregation in MCDA is based on DMs' preferences instead of relying on traditional cost-benefit analysis in which all criteria have to be converted to monetised measures (DETR, 1998).

Roy (1996) suggests three *problématiques* (fundamental problems) for MCDA, whereby a set of alternatives, **A**, is evaluated to produce a final decision result:

- *Choice*. Choose the best alternative from **A**.
- *Sorting*. Sort the alternatives of **A** into relatively homogeneous groups in a preference order.
- *Ranking*. Rank the alternatives of **A** from best to worst.

Among the above three types of decision problems, *ranking* produces the most comprehensive information with a full preference order of alternatives. Obviously, the best alternative (*choice*) can be conveniently identified if a full *ranking* is obtained. Also, a *sorting* problem can be addressed by applying a logical assignment procedure to the generated *ranking* results (Chen, Li, Kilgour, & Hipel, 2008). Various MCDA approaches are developed to handle different types of MCDA problems, including multiattribute utility theory (MAUT) (Keeney & Raiffa, 1976), outranking methods (Roy, 1996) and analytic hierarchy process (AHP) (Saaty, 1980), to name a few. A recent state-of-the-art review of MCDA (Figueira et al., 2005) summarizes a wide variety of MCDA approaches.

The TOPSIS (technique for order performance by similarity to ideal solution) method (Hwang & Yoon, 1981) constitutes a useful technique in solving ranking problems. The basic idea of the TOPSIS is simple and intuitive: measure alternatives' distances to predefined ideal and anti-ideal points first and, then, aggregate the separate distance information to reach overall evaluation results. Some features of TOPSIS, as summarized in Kim, Park, and Yoon (1997) and Shih, Shyur, and Lee (2007), include clear and easily understandable geometric meaning, simultaneously consideration from both best and worst points of view, and convenient calculation and implementation. Different methods have thus been developed to extend the original TOPSIS idea (Chen, 2000; Chen & Tzeng, 2004; Chu & Lin, 2009; Hwang, Lai, & Liu, 1993; Lai, Liu, & Hwang, 1994; Olcer, 2008; Shih et al., 2007; Wang & Lee, 2009).

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doi:10.1016/j.eswa.2010.10.039

The Ordered Weighted Averaging aggregation operators, commonly known as OWA operators, are introduced by Yager (1988) to provide a parameterized class of mean-type aggregation operators. Many notable mean operators, such as the Max, arithmetic average, median, and Min, are members of this class. OWA operators have been widely used in computational intelligence due to their flexibility in modeling linguistically expressed aggregation instructions (Cheng, Wang, & Wu, 2009). A comprehensive literature review and summary of OWA operators with diverse applications is provided in Torra and Narukawa (2007) and Yager and Kacprzyk (1997).

TOPSIS and OWA methods become increasingly popular research topic in several academic fields. For example, within the journal of *Expert Systems with Applications*, a search on the keywords "TOPSIS" through ScienceDirect identifies 105 papers. Especially there is a significant increase in 2009: over 80 papers has been published or accepted for publication. In this paper a hybrid approach of OWA aggregation and TOPSIS is designed to incorporate the unique features from both methods to provide additional flexibility for MCDA. The remainder of the paper is organized as follows: overviews of MCDA, OWA, and TOPSIS are given in Section 2; next, in Section 3 a hybrid method, integrating the OWA aggregation into the TOPSIS, is constructed and explained in detail; then, Section 4 presents a numerical example adapted from Shih et al. (2007) to demonstrate the proposed method and, finally, some concluding remarks are furnished in Section 5.

2. Overviews of MCDA, OWA and TOPSIS

2.1. An overview of MCDA

The analysis of an MCDA problem can be summarized as the following three steps (Chen, Kilgour, & Hipel, 2006): (1) *Problem construction*, in which the DM's objectives are defined, all possible alternatives are identified, and criteria are determined whereby successes in achieving the objectives are measured; (2) *Preference elicitation and aggregation*, in which the DM's preferences within and across criteria are obtained and aggregated; (3) *Implementation*, in which a constructed preference model is utilized to evaluate all alternatives, thereby the 'problématique' selected by the DM can be solved. The analysis results can be employed as an aid to the actual decision making process.

Step (1) aims to structure an MCDA problem. Let the set of alternatives be $\mathbf{A}=\{a^1,\ldots,a^i,\ldots,a^{|\mathbf{A}|}\}$ and the set of criteria be $\mathbf{C}=\{c_1,\ldots,c_j,\ldots,c_{|\mathbf{C}|}\}$, where $|\mathbf{X}|$ represents the cardinality of a set $\mathbf{X}$. When step (1) is completed, the consequence of alternative a^i on criterion c_j, denoted by m_j^i, will be measured for every $i=1,\ldots,|\mathbf{A}|$ and $j=1,\ldots,|\mathbf{C}|$, constituting the (i,j)-entry of a $|\mathbf{A}|\times|\mathbf{C}|$ matrix called the information (or performance) matrix. The structure of this matrix is shown in Fig. 1. Note that a consequence is a direct measurement of an alternative according to a criterion

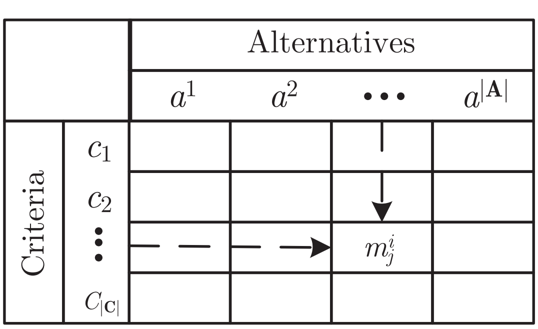

Fig. 1. Performance matrix in MCDA, adapted from Chen et al. (2006).

(e.g. cost in dollars). Generally speaking, a consequence is an objective physical measurement.

The DM's preferences are crucial in reaching a final recommendation for an MCDA problem, and different approaches to modeling preferences of the same problem may lead to different conclusions. Formally, as we interpret MCDA procedures, a DM may have preferences on consequences, called *values*, and preferences over criteria, referred to as *weights*.

Preferences on consequences, or "values," are refined data obtained by processing consequences (original and raw information) according to the needs and objectives of the DM. This is a necessary step to convert and normalize consequences into a common comparative ground as consequences on different criteria often assume significantly different formats. The general relationship between consequences and values can be expressed as a mapping from consequences to values, $v_j^i=f_j\left(m_j^i\right)$, where $v_j(a^i)$ and m_j^i are a value and a consequence measurement, respectively. The DM's values over all criteria for alternative a^i constitute a *value vector*, $\mathbf{v}(a^i)=(v_1(a^i),\ldots,v_{|\mathbf{C}|}(a^i))$. It is often assumed that criteria are preference monotonic along consequences: (1) *benefit criteria*: the larger the consequence value, the better; (2) *cost criteria*: the smaller the consequence value, the better.

Preferences on criteria, or "weights," refer to expressions of the relative importance of criteria. The weight for criterion $c_j\in\mathbf{C}$ is denoted by $w_j\in\mathbb{R}^+$. Usually it is required that $\sum_{j=1}^{|\mathbf{C}|}w_j=1$, and the DM's weight vector is denoted by $\mathbf{w}=(w_1,\ldots,w_j,\ldots,w_{|\mathbf{C}|})$.

After an MCDA problem is structured and preferences are obtained, a global model is required to aggregate preferences and solve the specified problématique. For $a^i\in\mathbf{A}$, the *overall evaluation* of alternative a^i is denoted by $V(a^i)\in\mathbb{R}$, where $V(a^i)=F(\mathbf{v}(a^i),\mathbf{w})$. Here, $F(\cdot)$ is a real-valued mapping from the value vector $\mathbf{v}(a^i)$ and the weight vector $\mathbf{w}$ to a numerical evaluation of a^i. A typical example is the *linear additive value function*, $V(a^i)=\sum_{j=1}^{|\mathbf{C}|}w_j\cdot v_j(a^i)$ (Hwang & Yoon, 1981).

2.2. OWA aggregation operators

An OWA operator is a process to aggregate a set of data, $\mathbf{B}=\{b^1,\ldots,b^{|\mathbf{B}|}\}$, into a representative datum, i.e. $\mathbb{R}^{|\mathbf{B}|}\to\mathbb{R}$, with an associated weight vector $\mathbf{Q}=(q_1,\ldots,q_{|\mathbf{B}|})$, ($|\mathbf{B}|=|\mathbf{Q}|$) such that $\sum_{j=1}^{|\mathbf{B}|}q_j=1,\ 0\leqslant q_j\leqslant 1$, and $OWA_{|\mathbf{Q}|}\left(b^1,\ldots,b^{|\mathbf{B}|}\right)=\sum_{i=1}^{|\mathbf{B}|}q_jb^{\sigma(j)}$, where $\{\sigma(1),\ldots,\sigma(|\mathbf{B}|)\}$ is a permutation of $\{1,\ldots,|\mathbf{B}|\}$ such that $b^{\sigma(j-1)}\geqslant b^{\sigma(j)}$, for all $j=\{2,\ldots,|\mathbf{B}|\}$, i.e. $b^{\sigma(j)}$ is the jth largest element in $\mathbf{B}$ (Torra & Narukawa, 2007). Hence, an important feature of OWA operators is the re-ordering of the elements that makes it a nonlinear operator, and the vector of $\mathbf{Q}$ is not the representation of relative importance of different types of information in $\mathbf{B}$, but a mechanism to smoothly achieve any kind of averaging between Max and Min for $\mathbf{B}$.

Two important features called the dispersion (entropy) and the "orness" are defined as $Disp(\mathbf{Q})=-\sum_{i=1}^{|\mathbf{B}|}q_j lnq_j$ and $orness(\mathbf{Q})=\frac{1}{|\mathbf{B}|-1}\sum_{j=1}^{|\mathbf{B}|}(|\mathbf{B}|-j)q_j$, respectively (Yager, 1988). The dispersion gauges the degree to which all data are equally aggregated. The orness is a value between 0 and 1 that represents the degree to which the aggregation is like an "OR" operation, and can be viewed as an optimism indicator of a decision maker. Some well-known averaging decision rules can be expressed as OWA operations below (Wang & Parkan, 2005):

- *OWA**: Set $\mathbf{Q}=(1,0,\ldots,0)$, then $OWA^*\left(b^1,\ldots,b^{|\mathbf{B}|}\right)=\max_{i=1}^{|\mathbf{B}|}(b^i)$, representing the most optimistic decision (maximax, "OR" decision) and $orness(\mathbf{Q})=1$;
- OWA_*: Set $\mathbf{Q}=(0,0,\ldots,1)$, then $OWA_*\left(b^1,\ldots,b^{|\mathbf{B}|}\right)=\min_{i=1}^{|\mathbf{B}|}(b^i)$, representing the most pessimistic decision (minimin, "AND" decision) and $orness(\mathbf{Q})=0$;