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Estimation the parameters of Lotka–Volterra model based on grey direct modelling method and its application

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ABSTRACT

In this paper, based on the grey direct modelling method, we present linear programming method to estimate the parameters of the Lotka–Volterra model under the criterion of the minimization of mean absolute percentage error (MAPE) (some authors called average relative error). Then use Lotka–Volterra model to analysis the relationship between two variables, and use discrete Lotka–Volterra model to forecast the two variables respectively, two practical examples are chosen for practical tests of this method, the results show that this method can provide empirical support for long-term qualitative analysis and obtain short-term quantitative prediction results, these have shown that this method is effective and applicable.

1. Introduction

The traditional GM(1,1) model is simple to calculate and has been successfully adopted in various fields (Deng, 1989; Liu & Lin, 2005), but it can not obtain higher forecasting precision, some scholars modified the GM(1,1) model, the main methods included discrete models (Xie & Liu, 2009), direct modelling method (Wang, 1988; Wang, 2003), first-entry GM(1,1) model (Tien, 2009), etc.

However, GM(1,1) model only indicates one variable, there are many variables in social system or economic system, there are complicated relationships among these variables, which interacts with each other, we must analysis the relationship among these variables. To address this problem, to date only the first-order N-variable grey model, GM(1,N) (Tien, 2009), is a multivariable grey model for multi-factor forecasting, but GM(1,N) can not reflect the relationship among these variables. Therefore, we use Lotka–Volterra model to analysis the relationship between two variables, then use discrete Lotka–Volterra model to forecast the two variables respectively.

One of the usual selection estimating method for parameters of forecasting model is the ordinary least-square method under the criterion of the minimization of the sum of squared errors (Deng, 1989; Tien, 2009, 2009; Liu & Lin, 2005; Wang, 1988, 2003; Xie & Liu, 2009), but we evaluate the forecasting model under the criterion of the minimization of the MAPE. We could improve on the above method if we minimize the MAPE, estimating parameters of forecasting model under the criterion of the minimization of the MAPE can reduce the MAPE. By doing so, the objective function is not-differentiable because of the presence of the absolute values, minimization of MAPE is computationally difficult. To address this problem, Wang and Hsu utilized genetic algorithm to estimate the parameters of GM(1,1) model under the criterion of the minimization of MAPE (Hsu, 2009; Wang & Hsu, 2008). Zhou et al. presented a new parameter optimization scheme of nonlinear grey Bernoulli model using the particle swarm optimization algorithm under the criterion of the minimization of MAPE (Zhou, Fang, Li, Zhang, & Peng, 2009).

All the above approaches are stochastic algorithms. In this paper, based on the grey direct modelling method, we propose a linear programming method in order to estimate parameters of Lotka–Volterra model under the criterion of the minimization of MAPE.

This paper is organized as follows, Section 2 provides an overview of the relevant literature on grey theory and Lotka–Volterra model. The linear programming method under the criterion of the minimization the MAPE is discussed in section 3. The numerical experiments are reported in Section 4. Concluding remarks is provided in the final section.

2. Literature review

2.1. Original GM(1,1) (Deng, 1989; Liu & Lin, 2005)

Assume that the sequence

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\},$$

is an original data sequence, where $x^{(0)}(k)$ is the time series data at time k, the sequence

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$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$,

is the accumulated generation sequence of X, set $x^{(1)}(1) = x^{(0)}(1)$, where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),$$

the equation

$$x^{(0)}(k) + az^{(1)}(k) = b,$$

is called a GM(1,1) model, where $z^{(1)}(k) = \frac{x^{(1)}(k)+x^{(1)}(k+1)}{2}$, $k = 1, 2, \ldots, n-1$. Use the ordinary least square method to estimate the parameters

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y,$$

where

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}, \quad B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix}.$$

Solve the whitenization equation $\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(k) = b$ of GM(1,1) model to obtain

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}.$$

Inverse accumulated generating operation is

$$\begin{aligned} x^{(0)}(k+1) &= x^{(1)}(k+1) - x^{(1)}(k) \\ &= \left[x^{(0)}(1) - \frac{b}{a}\right] e^{-ak}(1 - e^{a}), \quad k = 1, 2, \ldots, n-1. \end{aligned} \tag{1}$$

2.2. Grey Verhulst model (Liu & Lin, 2005)

$$X^{(0)} + aZ^{(1)} = b[Z^{(1)}]^2,$$

is called the grey Verhulst model. The least squares estimate of the parameters sequence a and b of the grey Verhulst model is given by

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y,$$

where

$$Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}, \quad B = \begin{pmatrix} -z^{(1)}(2) & [z^{(1)}(2)]^2 \\ -z^{(1)}(3) & [z^{(1)}(3)]^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & [z^{(1)}(n)]^2 \end{pmatrix},$$

the equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b[x^{(1)}]^2,$$

is called the whitenization equation of the grey Verhulst model. The solution of the Verhulst whitenization equation is given by

$$x^{(1)}(t) = \frac{ax^{(1)}(0)}{bx^{(1)}(0) + [a - bx^{(1)}(0)]e^{at}}.$$

The time response sequence of the grey Verhulst model is given by

$$\hat{x}^{(1)}(k+1) = \frac{ax^{(1)}(0)}{bx^{(1)}(0) + [a - bx^{(1)}(0)]e^{ak}}, \quad k = 1, 2, \ldots, n-1.$$

The Verhulst model is mainly used to study sigmoid processes, for example, it is often used in the prediction of biological growth, economic life span of consumable products, etc.

2.3. Direct modelling method of GM(1,1) (Wang, 1988, 2003)

Assume

$$X = \{x(1), x(2), \ldots, x(n)\},$$

is an original data sequence, where $x(k)$ is the time series data at time k, the grey derivative of the X is

$$dx(k) = x(k+1) - x(k),$$

the mean generated sequence of consecutive neighbors of X is

$$\frac{x(k) + x(k+1)}{2},$$

the equation

$$x(k+1) - x(k) + a\frac{x(k) + x(k+1)}{2} = b,$$

is an approximation of GM(1,1) model, where $k = 1, 2, \ldots, n-1$. Set $x(1) = x(1)$. Use the ordinary least square method to estimate the parameters

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (B^T B)^{-1} B^T Y,$$

where

$$B = \begin{pmatrix} x(2) - x(1) \\ x(3) - x(2) \\ \vdots \\ x(n) - x(n-1) \end{pmatrix}, \quad Y = \begin{pmatrix} \frac{x(2)+x(1)}{2} & 1 \\ \frac{x(3)+x(2)}{2} & 1 \\ \vdots & \vdots \\ \frac{x(n)+x(n-1)}{2} & 1 \end{pmatrix}.$$

Solve the grey difference equation $\frac{dx(t)}{dt} + ax(k) = b$ to obtain

$$\hat{x}(k) = \left[x(1) - \frac{b}{a}\right] e^{-ak} + \frac{b}{a}. \tag{2}$$

Substituting the

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix},$$

into Eq. (2), we obtain a non-homogenous exponential model

$$\hat{x}(k) = \left[x(1) - \frac{\hat{b}}{\hat{a}}\right] e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}, \quad k = 1, 2, \ldots, n-1.$$

Because direct modelling method of GM(1,1) is formulated by using the original data rather than the accumulated generation data, it need not inverse accumulated generating, we can see that Eq. (1) is an exponential model, Eq. (2) is a non-homogenous exponential model.

2.4. Lotka–Volterra model (Kim, Lee, & Ahn, 2006; Lee, Lee, & Oh, 2005; Tsai, 2009)

Lotka–Volterra model has been developed to model the interaction between the two competing species based on the logistic curve and extended on the analysis of technology diffusion in competitive or collaborative markets covered in relevant literature (Kim et al., 2006; Tsai, 2009; Lee et al., 2005). The Lotka–Volterra model of two species, X and Y, is as follows

$$\frac{dx}{dt} = a_1 x(t) - b_1 x(t)^2 - c_1 x(t) y(t), \tag{3}$$

$$\frac{dy}{dt} = a_2 y(t) - b_2 y(t)^2 - c_2 y(t) x(t), \tag{4}$$