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Solving the multi-stage portfolio optimization problem with a novel particle swarm optimization

Jun Sun [a,*], Wei Fang [a], Xiaojun Wu [a], Choi-Hong Lai [b], Wenbo Xu [a]

[a] School of Information Technology, Jiangnan University, No. 1800, Lihu Avenue, Wuxi, Jiangsu 214122, China
[b] School of Computing and Mathematical Sciences, University of Greenwich, Greenwich, London SE10 9LS, UK

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ABSTRACT

Solving the multi-stage portfolio optimization (MSPO) problem is very challenging due to nonlinearity of the problem and its high consumption of computational time. Many heuristic methods have been employed to tackle the problem. In this paper, we propose a novel variant of particle swarm optimization (PSO), called drift particle swarm optimization (DPSO), and apply it to the MSPO problem solving. The classical return-variance function is employed as the objective function, and experiments on the problems with different numbers of stages are conducted by using sample data from various stocks in S&P 100 index. We compare performance and effectiveness of DPSO, particle swarm optimization (PSO), genetic algorithm (GA) and two classical optimization solvers (LOQO and CPLEX), in terms of efficient frontiers, fitness values, convergence rates and computational time consumption. The experiment results show that DPSO is more efficient and effective in MSPO problem solving than other tested optimization tools.

1. Introduction

Financial optimization, involving asset allocation and risk management, is one of the most attractive areas in decision-making under uncertainty. While asset allocation problem decides the percentage of the overall portfolio value allocated to each portfolio component, risk management measures the risk of different investment instruments and creates or maintains portfolios with the specified risk-return characteristics. Although asset allocation and risk management are two indiscerptible parts in financial optimization, risk management has become a central topic for the management of financial institutions since 1990s (Elton, 1995). With the availability of a variety of sophisticated quantitative models and optimization tools, there is now a greater opportunity to manage risk more efficiently. Optimization models have made a significant impact on several dimensions of asset allocation and risk management. A multi-stage stochastic optimization is a quantitative model that integrates asset allocation strategies and saving strategies in a comprehensive fashion. It manages portfolio in constantly changing financial markets by periodically rebalancing the asset portfolio to achieve return maximization and risk minimization. The multi-stage optimization technique captures dynamic aspects of the problem, leading to optimal portfolio and efficient risk management. (Birge & Louveaux, 1997; Carino & Ziemba, 1998; Mulvey, Rosenbauma, & Shetty, 1997; Mulvey & Shetty, 2004; Mulvey & Vladimirou, 1995; Zhao & Zeimba, 2001). As such, it can provide superior performance over single period model (Berger & Mulvey, 1996; Carino et al., 1994).

The multi-stage portfolio optimization (MSPO) problem is complex and non-linear with many local optima. A number of different algorithmic approaches have been proposed for solving stochastic optimization problems. To solve the MSPO problem, one may employ linear programming solvers such as CPLEX and OSL, by which the nonlinear terms in the objective function can be piecewise linearized (Danzig & Infanger, 1993). The interior-point algorithms, employed by LOQO solver, are well suited to the scenario structure of multi-stage stochastic programs (Vanderbei, 1992). Searching the global solution by these methods, however, is computationally expensive and ineffectively.

Since time is a constraint for financial problems, many researchers have employed heuristic methods to find optimal asset allocation in order to achieve a trade-off between the performance and the computational time. Berger, Glover, and Mulvey (1995), Berger and Mulvey (1996) applied Tabu Search, an adaptive memory programming, to the problem. Their method improves computational performance considerably as compared with interior-point methods for solving the problem. Chan et al. use genetic algorithms (GA) as their self-learning portfolio optimizer to optimize one's asset allocation in their portfolio optimization system (Chan, Wong, Cheung, & Tang, 2002). GA reassures a higher chance of reaching a global optimum by starting with multiple random

* Corresponding author. Tel./fax: +86 510 85912136.
E-mail address: sunjun_wx@hotmail.com (J. Sun).

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search points and considering several candidate solutions simultaneously. It can be used to solve difficult problems with objective functions that do not posses "nice properties" such as continuity, differentiability, satisfaction of the Lipshcitze Condition, etc.

Although GA has many advantages over traditional local optimization algorithms, it is a time consuming solver because of its slow convergence speed (Fogel, 1994; Michalewicz, 1992). In 1990s, a development in optimization theory saw the emergence of swarm intelligence, a category of stochastic search methods for solving global optimization (GO) problems (Dorigo, Maniezzo, & Colorni, 1996; Kennedy & Eberhart, 2001). particle swarm optimization (PSO) method is one of its member. It was originally proposed by Kennedy and Eberhart as a simulation of social behavior of bird flock, and initially introduced as an optimization method in 1995 (Kennedy & Eberhart, 1995). The PSO algorithm can be easily implemented and is computationally inexpensive, since its memory and CPU speed requirements are low. Moreover, it does not require gradient information of the objective function under consideration but only its values, and it uses only primitive mathematical operators. PSO has been proved to be an efficient method for many GO problems and in some cases it does not suffer the difficulties encountered by GA (Angeline, 1998a; Eberhart & Shi, 1998).

In this paper, inspired by the motion of electrons in a conductor under electric field, we propose a new variation of PSO, called drift particle swarm optimization (DPSO), and apply it to multi-stage portfolio optimization (MSPO) problem. DPSO has some characteristics of the PSO algorithm, such as collectiveness and mutual learning among individuals. However, unlike PSO, DPSO is a global convergent algorithm and has stronger search ability than PSO. The efficiency and effectiveness of DPSO in the MSPO problem solving was evaluated by testing the algorithm on the problems with sample data collected from S&P 100 Index and the prices of its component stocks. PSO, GA, CPLEX, and LOQO were also tested for the purpose of performance comparison.

The rest of the paper is organized as follows. In Section 2, the MSPO model is described. The proposed DPSO algorithm is presented in Section 3. Section 4 presents how to apply DPSO in the MSPO problem. The experiment results are provided in Section 5. Some concluding remarks are given in Section 6.

2. Multi-stage portfolio optimization model

2.1. Problem statement

The multi-stage portfolio optimization (MSPO) model views the financial optimization as a multi-period dynamic problem where transactions take place at discrete time points. To define the model, we divide the entire planning horizon T into two discrete intervals T_1 and T_2, where $T_1 = 0,1,\ldots,\tau$ and $T_2 = \tau + 1,\ldots,T$. The former corresponds to periods in which investment decisions are made. Period τ defines the date of planning horizon and we focus on the investor's position at the beginning of period τ. Decisions occur at the beginning of each time stage. Much flexibility exists so that an active trader might see his time interval as short as minutes, whereas a pension plan advisor will be more concerned with much longer planning periods such as the dates between the annual Board of Director's meetings. It is possible for the steps to vary over time-short intervals at the beginning of planning period and longer intervals towards the end. T_2 handles the horizon at time τ by calculating economic and other factors beyond period τ up to period T. The investor cannot render any active decisions after the end of period τ.

Asset investment categories are defined by a subscript set $A = \{1,2,\ldots,I\}$, with category 1 representing cash. The remaining categories can include broad investment groupings such as stocks, bonds, and real estate. The categories should track well-defined market segment. Ideally, the co-movements between pairs of asset returns would be relatively low so that diversification can be done across the asset categories.

Uncertainty is modeled through a large but finite number S of scenarios. Each scenario represents a possible realization of all uncertain parameters in the model. To be specific, let ω_t represent the vector of random parameters whose values are revealed in period t. Then the set of all scenarios is the set of all realizations $(\omega_1^s, \omega_2^s, \ldots, \omega_\tau^s),\ s \in S := \{1,2,\ldots,S\}$, of $(\omega_1, \omega_2, \ldots, \omega_\tau)$. Each scenario s has a probability π_s, where $\pi_s > 0$ and $\sum_{s=1}^{S} \pi_s = 1$. Since in a dynamic model information on actual value of the uncertain parameters is revealed in stages, a suitable representation of scenarios is given by a scenario tree. Fig. 1 shows a scenario tree when $\tau = 3$ and $S = 8$. Each path from $t = 0$ to $t = \tau$ represents one scenario. Any node of the tree, corresponding to time t, symbolizes a possible state of the world at time t, represented by the observed values of $\omega_1, \omega_2, \ldots, \omega_t$. The branches directly to the right of it symbolize the various values of ω_{t+1} (and their corresponding conditional probabilities) given the realization of $\omega_1, \omega_2, \ldots, \omega_t$. Obviously, all scenarios passing this node have the same history in periods $1,2,\ldots,t$. The status of decision variables is related to the scenario tree too. Basically, a decision at time t may depend on the observed part of the scenario at that time, but not on unknown values of future periods. That is, for each possible history (i.e. for each node at time t in the scenario tree) there is precisely one vector of decision variables representing the decisions at hand.

We assume that the portfolio is rebalanced at the beginning of each period. Alternatively, we could simply make no transaction except reinvest any dividend and interest – a buy and hold strategy. For convenience, we also assume that the cash flows are reinvested in the generating asset category and all the borrowing is done on a single period basis.

2.2. Parameters and decision variables

For each $n \in A$, $t \in T_1$ and $s \in S$, we define the following parameters and decision variables.

Parameters

$r_{n,t}^s$ $= 1 + \rho_{n,t}^s$, where $\rho_{n,t}^s$ is the return percentage of asset n, time period t under scenario s (for example, projected by the stochastic scenario generator (Mulvey, Rosenbauma, & Shetty, 1996, 1999))

π_s probability that scenario s occurs, thus $\sum_s^S \pi_s = 1$

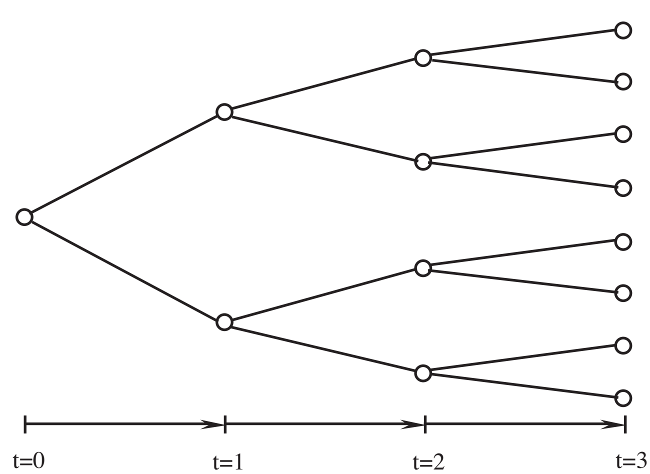

Fig. 1. A scenario tree with two scenarios and three time periods.