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Best-worst multi-criteria decision-making method: Some properties and a linear model ☆

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A B S T R A C T

The Best Worst Method (BWM) is a multi-criteria decision-making method that uses two vectors of pairwise comparisons to determine the weights of criteria. First, the best (e.g. most desirable, most important), and the worst (e.g. least desirable, least important) criteria are identified by the decision-maker, after which the best criterion is compared to the other criteria, and the other criteria to the worst criterion. A non-linear minmax model is then used to identify the weights such that the maximum absolute difference between the weight ratios and their corresponding comparisons is minimized. The minmax model may result in multiple optimal solutions. Although, in some cases, decision-makers prefer to have multiple optimal solutions, in other cases they prefer to have a unique solution. The aim of this paper is twofold: firstly, we propose using interval analysis for the case of multiple optimal solutions, in which we show how the criteria can be weighed and ranked. Secondly, we propose a linear model for BWM, which is based on the same philosophy, but yields a unique solution.

1. Introduction

In general, decision-making can be defined as identifying and selecting an alternative from a set of alternatives based on the preferences of the decision-maker(s). In most cases, several criteria are involved in this identification and selection process, which is why these problems are called multi-criteria decision-making problems. Different decision-makers value the criteria involved differently. In the past decades, several multi-criteria decision-making methods have been proposed to help decision-makers find the values of the criteria and the alternatives based on their preferences. As the aim of this paper is not to review these methods, we refer the readers to some textbooks that cover the most commonly used MCDM methods [1–3], and some review papers [4,5]. One of the most recently developed methods is the best worst method (BWM) [6], which is a comparison-based method that conducts the comparisons in a particularly structured way, such that not only is less information is required, but the comparisons are also more consistent. In some cases, BWM results in multi-optimality, which means that solving the problem results in different sets of weights for the criteria. This feature of the method may be desirable in some cases. For instance, when debating has an important role in the decision-making process [7] (e.g. political decision-making), multi-optimality provides the decision-makers with the freedom to incorporate higher-level information (information that cannot be modeled) into their decision-making process. In other cases, however, the decision-maker may prefer a unique solution (e.g. when there is no debating or when there is no higher-level information that needs to be considered). The main contribution of this paper is twofold. We first ascertain some solution properties of BWM and show how we can determine the ranges of the weights of different criteria in the case of multi-optimality. We then use interval analysis as a way to analyze such cases and to determine the ranking of the criteria. Secondly, we propose a linear BWM, based on the same philosophy of BWM, that always results in unique solution.

In the next section, we provide an overview of the BWM, after which we discuss the multi-optimality property of this method in Section 3. Next, we describe the interval analysis and incorporate it in the method. Numerical examples are used to illustrate the procedure we propose to rank the interval weights. In Section 4, we propose a linear model of BWM and also solve some examples for this model. The conclusions are presented in Section 5.

2. Best worst method

Here, we briefly describe the steps of BWM that can be used to derive the weights of the criteria [6].

Step 1. Determine a set of decision criteria.

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In this step, the decision-maker identifies n criteria $\{c_1, c_2, \cdots, c_n\}$ that are used to make a decision.

Step 2. Determine the best (e.g. most desirable, most important) and the worst (e.g. least desirable, least important) criteria.

Step 3. Determine the preference of the best criterion over all the other criteria, using a number between 1 and 9. The resulting best-to-others (BO) vector would be:

$$A_B = (a_{B1}, a_{B2}, \ldots, a_{Bn}),$$

where a_{Bj} indicates the preference of the best criterion B over criterion j. It is clear that $a_{BB} = 1$.

Step 4. Determine the preference of all the criteria over the worst criterion, using a number between 1 and 9. The resulting others-to-worst (OW) vector would be:

$$A_W = (a_{1W}, a_{2W}, \ldots, a_{nW})^T,$$

where a_{jW} indicates the preference of the criterion j over the worst criterion W. It is clear that $a_{WW} = 1$.

Step 5. Find the optimal weights $(w_1^*, w_2^*, \ldots, w_n^*)$.

The aim is to determine the optimal weights of the criteria, such that the maximum absolute differences $\left|\frac{w_B}{w_j} - a_{Bj}\right|$ and $\left|\frac{w_j}{w_W} - a_{jW}\right|$ for all j is minimized, which is translated to the following minmax model:

$$\begin{aligned}
&\min \max_j \left\{ \left|\frac{w_B}{w_j} - a_{Bj}\right|, \left|\frac{w_j}{w_W} - a_{jW}\right| \right\} \\
&\text{s.t.} \\
&\sum_j w_j = 1 \\
&w_j \geq 0, \quad \text{for all } j
\end{aligned} \tag{1}$$

Model (1) is equivalent to the following model:

$$\begin{aligned}
&\min \xi \\
&\text{s.t.} \\
&\left|\frac{w_B}{w_j} - a_{Bj}\right| \leq \xi, \quad \text{for all } j \\
&\left|\frac{w_j}{w_W} - a_{jW}\right| \leq \xi, \quad \text{for all } j \\
&\sum_j w_j = 1 \\
&w_j \geq 0, \quad \text{for all } j
\end{aligned} \tag{2}$$

For any value of ξ, multiplying the first set of the constraints of model (2) by w_j and the second set of constraints by w_W, it can be seen that the solution space of model (2) is an intersection of $4n-5$ linear constraints ($2(2n-3)$ comparison constraints and one constraint for the weights sum), thus given a large enough ξ that the solution space is non-empty. Solving model (2), the optimal weights $(w_1^*, w_2^*, \ldots, w_n^*)$ and ξ^* are obtained.

According to [6], a consistent comparison is defined as follows:

Definition 1. A comparison is fully consistent when $a_{Bj} \times a_{jW} = a_{BW}$, for all j, where a_{Bj}, a_{jW} and a_{BW} are respectively the preference of the best criterion over the criterion j, the preference of criterion j over the worst criterion, and the preference of the best criterion over the worst criterion.

Table 1 shows the maximum values of ξ(consistency index) for different values of a_{BW}.

Table 1
Consistency Index (CI) Table [6].

a_{BW}	1	2	3	4	5	6	7	8	9
Consistency Index (max ξ)	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

Table 2
Best-to-others (BO) and others-to-worst (OW) pairwise comparison vectors: Example 1.

BO	*Quality*	*Price*	*Comfort*	*Safety*	*Style*
Best criterion: *price*	2	1	4	2	8
OW				Worst criterion: *style*	
Quality				4	
Price				8	
Comfort				2	
Safety				4	
Style				1	

Table 3
Best-to-others (BO) and others-to-worst (OW) pairwise comparison vectors: Example 2.

BO	*Quality*	*Price*	*Comfort*	*Safety*	*Style*
Best criterion: *price*	2	1	4	3	8
OW				Worst criterion: *style*	
Quality				4	
Price				8	
Comfort				2	
Safety				3	
Style				1	

Table 4
Best-to-others (BO) and others-to-worst (OW) pairwise comparison vectors: Example 3.

BO	*Quality*	*Price*	*Comfort*	*Safety*	*Style*
Best criterion: *price*	2	1	4	3	8
OW				Worst criterion: *style*	
Quality				4	
Price				8	
Comfort				4	
Safety				2	
Style				1	

Considering the consistency index (Table 1), the consistency ratio is calculated as follows:

$$\textit{Consistency Ratio} = \frac{\xi^*}{\textit{Consistency Index}} \tag{3}$$

Consistency Ratio $\in [0, 1]$, values close to 0 show more consistency, while values close to 1 show less consistency.

The solution space of (2) includes all the positive values for w_j, $j = 1, \ldots, n$, such that the sum of weights be 1 and the violation of all the weight ratios from their corresponding comparison be at most ξ. Here we show that model (2) might result in multiple optimal solutions for problems with more than three criteria.

Suppose that for a problem with n criteria (weight variables), we have ξ^*. Replacing ξ by ξ^* in the right-hand side of the constraints of (2), the optimal solution would be the results of the following linear system:

$$\begin{cases}
|w_B - a_{Bj} w_j| \leq \xi^* w_j, & \text{for all } j \\
|w_j - a_{jW} w_W| \leq \xi^* w_W, & \text{for all } j \\
\sum_j w_j = 1 & \\
w_j \geq 0, & \text{for all } j
\end{cases} \tag{4}$$