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A note on "Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract"☆

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ABSTRACT

In this short note, we first improve the proof in Zhang et al. [1] to show the strict concavity of the unit time total profit of the whole supply chain with respect to preservation technology investment without approximation. We then generalize the model of Zhang et al. [1] to a broader class of market demand functions. Additionally, theoretical results are provided to illustrate the features of the proposed model.

1. Introduction

Deterioration is a common phenomenon in daily life caused by poor storage and preservation, and erodes profit margins. Deteriorating inventory models have been extensively studied since the first mathematical formulation was introduced by Ghare and Schrader [2]. However, previous studies have often treated the deterioration rate of goods as an exogenous variable. In practice, businesses can slow the deterioration process by investing in preservation technology. Accordingly, Hsu et al. [3] investigated how preservation technology affects an exponentially decaying inventory model involving partial backorders. Dye [4] extended the model of Hsu et al. [3] to a non-instantaneous deteriorating inventory system. In addition, Dye [4] demonstrated that increasing preservation technology investment improves customer service. Because pricing is the main strategy used by retailers to maximize profit, He and Huang [5] and Zhang et al. [6] have considered an optimal prevention technology investment and lot-sizing problem for deteriorating items with price-sensitive demand. Tsao [7] extended the model of Dye [4] to consider a joint location and preservation technology investment decision-making problem for non-instantaneous deteriorating items in a trade credit context.

Recently, to develop a more practical inventory model, Zhang et al. [1] considered a deteriorating inventory management problem with price-dependent demand in a two-echelon supply chain comprising one manufacturer and one retailer. In the model, both the manufacturer and retailer are assumed to cooperatively invest in preservation technology to reduce the deterioration rate. Zhang et al. [1] developed the joint pricing and preservation technology investment strategy for two supply chain scenarios. The first is the integrated scenario, the goal in which is to maximize the sum of manufacturer and retailer profits. The second is the decentralized scenario, in which the manufacturer selects the wholesale price and subsidy proportion first, and then the retailer observes this decision and determines its own retail price and preservation technology investment. Furthermore, they also designed a revenue sharing and cooperative investment contract for facilitating supply chain coordination. However, they proved the concavity of the unit time total profit of the whole supply chain Π_{sc} with respect to preservation technology investment u in the integrated scenario by employing the first-order Taylor approximation (i.e., $\ln(1-x) \approx -x$ as $x \to 0$). Moreover, the market demand in Zhang et al. [1] is assumed to be a specific function of the retail price, limiting the applicability of the study.

In this short note, to obtain more robust and generalizable results, we first improve the proof in Zhang et al. [1] to show the strict concavity of Π_{sc} with respect to u without Taylor approximation. We then generalize the model of Zhang et al. [1] to a broader class of market demand functions. Theoretical results are also provided to illustrate the features of the proposed model. The appendix contains all proofs.

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2. Model and theoretical results

For easy tractability with Zhang et al. [1], we use the same notation and assumptions. The unit time total profit for the retailer and manufacturer constructed by Zhang et al. [1] can be summarized as follows:

$$\Pi_r=(p+c_d)D(p)-\frac{(c_d+w)\left\{e^{\theta[1-f(u)]T}-1\right\}D(p)}{\theta[1-f(u)]T}$$
$$-\frac{h\left\{e^{\theta[1-f(u)]T}-1\right\}D(p)}{\theta^2[1-f(u)]^2T}+\frac{hD(p)}{\theta[1-f(u)]}-\frac{K_r}{T}-(1-\phi)gu \quad (1)$$

and

$$\Pi_m=\left\{w+c_d+\frac{h}{\theta[1-f(u)]}\right\}\frac{q}{\theta[1-f(u)]T}\left\{1-e^{\theta[1-f(u)](t_m-T)}\right\}$$
$$+\left\{c_p+c_d+\frac{h}{\theta[1-f(u)]}\right\}\frac{q(t_m-T)}{T}-\frac{K_m}{T}-\phi gu, \quad (2)$$

where

$$t_m=T+\frac{\ln\left\{1-\frac{\left\{e^{\theta[1-f(u)]T}-1\right\}D(p)}{q}\right\}}{\theta[1-f(u)]}. \quad (3)$$

To generalize the model of Zhang et al. [1], we make the following assumptions about the market demand rate.

Assumption 1. Let p be the retail price charged by the retailer. The market demand is given by the demand function $D(p)$, which is assumed to be positive, bounded, strictly decreasing and continuously differentiable (i.e., $0<D(p)<\infty$ and $D'(p)<0$ for $p\in(0,\overline{p})$, where $\overline{p}$ is the maximal retail price of the product). Moreover, we assume that $1/D(p)$ is strictly convex in p and $\lim_{p\to\overline{p}}pD(p)=0$.

Assumption 2. Denote $\underline{p}$ as the minimum retail price of the product. We assume that $D(\underline{p})<q$. Furthermore, because c_p and w represent the unit production cost of the manufacturer and wholesale price charged by the manufacturer, respectively, without loss of generality, we assume that $\underline{p}>w>c_p>0$.

Note that the strict convexity of $1/D(p)$ in Assumption 1 implies that the gross profit $\pi(p)\equiv(p-w)D(p)$ is strictly pseudoconcave in p. To illustrate this, by substituting the result of $\mathrm{d}\pi(p)/\mathrm{d}p=(p-w)D'(p)+D(p)=0$ into $\mathrm{d}^2\pi(p)/\mathrm{d}p^2=(p-w)D''(p)+2D'(p)$, we obtain $\mathrm{d}^2\pi(p)/\mathrm{d}p^2=2\left[D'(p)\right]^2-D(p)D''(p)/D'(p)$. Because the strict convexity of $1/D(p)$ is equivalent to $2\left[D'(p)\right]^2-D(p)D''(p)>0$, $\pi(p)$ is strictly concave at any interior critical point, indicating that $\pi(p)$ is strictly pseudoconcave in p. The condition $\lim_{p\to\overline{p}}pD(p)=0$, and considering $\mathrm{d}\pi(p)/\mathrm{d}p|_{p=w}=D(w)>0$ and $\pi(w)=0$, implies that $\pi(p)$ is unimodal in p and has a finite interior maximizer in $(0,\overline{p})$. Furthermore, let $\eta\equiv D(p)D''(p)/D'(p)^2$ denote a measure of the curvature of demand function, then demand is convex if $\eta>0$, and concave otherwise. Because $1/D(p)$ is strictly convex in p, it is easily to see that $\eta<2$. Assumption 1 indicates that the curvature of $D(p)$ should not be highly positive, that is, the demand function is not too convex to the origin. Therefore, Assumption 1 provides a weaker condition than concavity of $D(p)$ with respect to p to ensure the existence and uniqueness of optimal price to maximize the gross profit.

Taking the derivative of t_m with respect to u yields

$$\frac{\partial t_m}{\partial u}=\left\{\frac{TD(p)e^{\theta[1-f(u)]T}}{q[1-f(u)]\left\{1-\frac{\{e^{\theta[1-f(u)]T}-1\}D(p)}{q}\right\}}+\frac{\ln\left\{1-\frac{\{e^{\theta[1-f(u)]T}-1\}D(p)}{q}\right\}}{\theta[1-f(u)]^2}\right\}f'(u).$$

Let $x=\theta[1-f(u)]T$ and $y=(e^x-1)D(p)/q$. Because $xe^x-(e^x-1)=\sum_{n=1}^{\infty}(n-1/n!)x^n>0$ and $(y/1-y)+\ln(1-y)=\sum_{n=1}^{\infty}(n-1/\ n)y^n>0$ for all $x>0$ and $0<y<1$, we obtain

$$\frac{\partial t_m}{\partial u}=\frac{1}{\theta[1-f(u)]^2}\left\{\frac{xe^x}{1-\frac{(e^x-1)D(p)}{q}}\frac{D(p)}{q}+\ln\left\{1-\frac{(e^x-1)D(p)}{q}\right\}\right\}$$

$$f'(u)\geq\frac{1}{\theta[1-f(u)]^2}\left\{\frac{(e^x-1)\frac{D(p)}{q}}{1-\frac{(e^x-1)D(p)}{q}}+\ln\left\{1-\frac{(e^x-1)D(p)}{q}\right\}\right\}$$

$$f'(u)=\frac{f'(u)}{\theta[1-f(u)]^2}\sum_{n=1}^{\infty}\frac{n-1}{n}y^n>0,$$

which implies that t_m increases strictly as u increases. Recall that $f(u)$, $0\leq f(u)\leq 1$, represents the proportion of reduced deterioration rate, $f'(u)>0$ and $\lim_{u\to\infty}f(u)=1$, it is straightforward to observe that $\lim_{u\to\infty}\ln\left[1-\frac{\{e^{\theta[1-f(u)]T}-1\}D(p)}{q}\right]=0$ and $\lim_{u\to\infty}\theta[1-f(u)]=0$. We can here use the L' Hospital rule to verify that

$$\lim_{u\to\infty}\frac{\ln\left[1-\frac{\{e^{\theta[1-f(u)]T}-1\}D(p)}{q}\right]}{\theta[1-f(u)]}=\frac{D(p)}{q}T.$$

Therefore, Assumption 2 ensures that the supply chain can continue to function smoothly.

2.1. Integrated scenario

In the integrated scenario, the objective of the supply chain is to determine the optimal retail pricing and preservation technology investment strategies, denoted by Π_{sc}, which maximize the sum of manufacturer and retailer profits. Hence, the problem can be formulated as

$$\max_{\underline{p}\leq p\leq\overline{p},u\geq 0}\quad \Pi_{sc}=\Pi_r+\Pi_m=(p+c_d)D(p)$$
$$+\frac{hD(p)}{\theta[1-f(u)]}+\left\{c_p+c_d+\frac{h}{\theta[1-f(u)]}\right\}\frac{q(t_m-T)}{T}$$
$$-\frac{K_m+K_r}{T}-gu. \quad (4)$$

In this subsection, we first improve the proof in the model of Zhang et al. [1] to show the strict concavity of Π_{sc} with respect to u without Taylor approximation. The following lemma states that a unique preservation technology investment $u\in[0,\infty)$ exists such that the unit time total profit of the whole supply chain Π_{sc} is maximized for a given retail price p.

Lemma 1. *For a given p, if the law of diminishing marginal productivity of invested capital holds true (i.e., $f'(u)>0$ and $f''(u)<0$), then a unique $u\in[0,\infty)$ exists such that Π_{sc} is maximized.*

By using Lemma 1, we can show the existence of equilibrium in the integrated scenario. The following theorem formally states this result.

Theorem 1. *If the conditions in Lemma 1 are met, at least one local maximizer of Π_{sc} exists for the maximization problem.*

Theorem 1 indicates that the object of the supply chain in the integrated scenario has at least one global solution. If an interior solution exists, according to the Envelope theorem, the first-order condition for the optimality of p^* is $\mathrm{d}\Pi^*_{sc}(p,u(p))/\mathrm{d}p=0$. However, determining the closed-form expressions for p and u is difficult because of the structure of the nonlinear system. To solve the problem numerically by using a fairly iterative search algorithm, we must prove the uniqueness of the optimal retail price p to Π_{sc} for any given preservation technology investment u. By applying