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Cost decompositions and the efficient subset ☆

Rolf Färe [a], Hirofumi Fukuyama [b,*], Shawna Grosskopf [c], Valentin Zelenyuk [d]

[a] Oregon State University, USA
[b] Fukuoka University, Japan
[c] Oregon State University, USA and CERE, Umea, Sweden
[d] School of Economics & Australian Institute for Business and Economics (AIBE) & Centre for Efficiency and Productivity Analysis (CEPA), The University of Queensland, Australia

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ABSTRACT

This paper develops two cost decompositions based on the multiplicative Russell and additive slack-based (in)efficiency measurement frameworks. While the multiplicative cost decomposition is a straightforward extension of the standard cost decomposition, the decomposition we develop in this paper incorporates slacks directly so that efficiency is measured relative to the efficient subset. To show the applicability of our novel approach, we provide an illustration using a data set used in the literature.

1. Introduction

The measurement of input technical efficiency relative to the efficient subset of an input set goes back to Färe [14] who proposed minimizing inputs one at a time, i.e., nonradially. Later, Färe and Lovell [20] proposed what they called the Russell measure (also referred to by others as the Färe-Lovell measure) which was also nonradial but summed over the individual input inefficiency components. These measures eliminated all technical inefficiencies including those due to 'slacks' as opposed to the radial Farrell (1957) measure of input technical efficiency which uses the isoquant rather than the efficient subset as the reference for technical efficiency. Thus, when the efficient subset differs from the isoquant, radial measures of technical efficiency such as the Farrell measure and nonradial measures may differ. Furthermore this may affect not only technical efficiency but allocative efficiency as well, resulting in different decompositions of the overall (e.g., cost or revenue) efficiency. This discrepancy is the motivation for considering nonradial measures as part of a decomposition of the overall Farrell measure of cost or revenue efficiency. The first such result was obtained by Färe, Grosskopf and Zelenyuk [19]. Their decomposition comes from introducing a multiplicative version of the Russell measure; and here we expand on their result. In this paper we will focus on the cost efficiency or input orientation, but similar decompositions can be developed for revenue efficiency as well.

The introduction of the directional distance functions[1], by Chambers, Chung and Färe [7,8], is another alternative nonradial (additive) way of estimating technical (in)efficiency. In fact, the directional input distance function may be turned into a slack-based additive efficiency measure[2]. We can identify two classes of nonradial slack-based technical efficiency measures, a multiplicative and an additive measure, both with an indication property such that the multiplicative (additive) measure equals one (zero) if and only if the input vector belongs to the efficient subset. The efficient subset is particularly important in efficiency measurement because input vectors in the efficient subset cannot be reduced without decreasing at least one input and/or increasing at least one output. On the other hand, if the input vector is in the isoquant but not in the efficient subset, then it is possible to reduce at least one input given a fixed level of outputs. Whereas the Farrell measure and the directional input distance function are constructed relative to an isoquant, the slack-based measures are constructed relative to the efficient subset. Consequently, it is of great interest to develop efficiency analysis based on a slack-based efficiency measurement framework.

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* Corresponding author.

[1] The directional distance functions are production counterparts of Luenberger's shortage function which is based on the utility function and the consumption possibility set. See Luenberger [27]. The directional input distance function is a production counterpart of Luenberger's [26] benefit function developed in a consumer context.

[2] See Färe and Grosskopf [16].

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In this paper we introduce both an additive and a multiplicative slack-based approach for the decomposition of the Farrell cost efficiency measure. The rest of the paper unfolds as follows. While Section 2 describes the basics, Section 3 introduces a multiplicative cost decomposition and its corresponding allocative efficiency measure based on the multiplicative Russell measure developed by Färe, Grosskopf and Zelenyuk [19]. Section 4 introduces a new cost decomposition based on the additive Russell measure. Section 5 extends the additive approach into data envelopment analysis (DEA) and provides an empirical illustration using a real-life data set documented in Banker and Maindiratta [4]. The last section gives a brief summary.

2. Background and methodology

In this section, we outline the theoretical background for our paper. Then we develop the methodology for it by building on the inequality of Mahler [28]. Let $x \in \Re^N_+$ be an input vector and $y \in \Re^M_+$ be an output vector. The input requirement sets are defined as

$$L(y) = \{x: \ x \text{ can produce } y\}, \quad y \in \Re^M_+ \tag{1}$$

and are our representation of the technology, which is assumed to be a nonempty, closed, strongly disposable set satisfying the boundedness of $\{y: \ x \in L(y)\}$ as well as no free lunch and convexity of $L(y)$. For the details of these regularity conditions, see for example [21]. Technology can equivalently be expressed `as the production possibility set $T = \{(x,y): \ x \in L(y)\}$, i.e., $x \in L(y) \Leftrightarrow (x,y) \ \in T$. The following two subsets of (1) are important for the paper. The isoquant for $y \in \mathbb{R}^M_+$ is defined as

$$IsoqL(y) = \{x : x \in L(y) \text{ and if } \lambda < 1 \text{ then } \lambda x \notin L(y)\}$$

and the efficient subset is [3]

$$EffL(y) = \{x : x \in L(y) \text{ and if } x' \leq x \text{ then } x' \notin L(y)\}.$$

Clearly, $EffL(y) \subseteq IsoqL(y)$. Shephard's [31] input distance function is defined as

$$D_i(y,x) = \sup \quad \{\lambda: \ x/\lambda \in L(y)\}$$

which characterizes the production technology (1). Now assume that input prices $w \in \mathbb{R}^N_{++}$ are given, then the cost function is

$$C(y,w) = \min_x \{w \cdot x : x \in L(y)\}$$

where $w \cdot x$ is the inner product, i.e., $w \cdot x = \sum_{n=1}^N w_n x_n$.

From the Mahler [28] inequality, we have

$$\frac{C(y,w)}{w \cdot x} \leqq \frac{1}{D_i(y,x)}$$

where $C(y,w)/w \cdot x$ is referred to as the cost efficiency measure and $1/D_i(y,x)$ is called the technical (Farrell) input oriented efficiency measure. An allocative efficiency measure, call it $AE(y,x,w)$, is then defined as the multiplicative residual required to close the inequality, so that

$$\frac{C(y,w)}{w \cdot x} = \frac{1}{D_i(y,x)} \times AE(y,x,w) \tag{2}$$

which is sometimes referred to as the Farrell decomposition of cost efficiency.

Throughout the paper, we use the above approach:

i. Start with the cost inequality.
ii. Derive a technical efficiency measure (input oriented).
iii. Complete the decomposition by introducing an allocative efficiency measure (input oriented).

[3] "$\leq$" means "$\leqq$ but $\neq$".

3. The multiplicative approach[4]

The Russell measure (RM) has the indication property that it yields unity (or 100%) if and only if the input vector belongs to the efficient subset $EffL(y)$. Here we follow Färe, Grosskopf and Zelenyuk [19] and define the multiplicative Russell measure as

$$RM^{mult}(y,x) = \min_{\lambda_1,\dots,\lambda_N} \left\{ \left(\prod_{n=1}^N \lambda_n \right)^{1/N} : \begin{array}{l} (\lambda_1 x_1, \dots, \lambda_N x_N) \in L(y), \\ 0 < \lambda_n \leqq 1, \quad n = 1, \dots, N \end{array} \right\}. \tag{3}$$

This definition differs from the additive (original) Russell measure [20], which we denote as $RM^{add}(y,x)$, whose objective function was additive, i.e.,

$$\frac{\sum_{n=1}^N \lambda_n}{N}.$$

For the rest of the paper, we assume that x is strictly positive, i.e., $x_n > 0, \quad n = 1, \dots, N$.

We note that

$$RM^{mult}(y,x) = 1 \quad \text{if and only if} \quad x \in EffL(y)$$

and that

$$(\lambda_1^* x_1, \dots, \lambda_N^* x_N) \in L(y)$$

where $\lambda_n^*, \quad n = 1, \dots, N$, are the optimizers of (3). Together with the cost inequality we get

$$C(y,w) \leq w_1 \lambda_1^* x_1 + \dots + w_N \lambda_N^* x_N$$

$$= RM^{mult}(y,x) \left(\frac{\lambda_1^* w_1 x_1}{\left(\prod_{n=1}^N \lambda_n^* \right)^{1/N}} + \dots + \frac{\lambda_N^* w_N x_N}{\left(\prod_{n=1}^N \lambda_n^* \right)^{1/N}} \right)$$

and multiplying both sides with $1/w \cdot x$ yields

$$\frac{C(y,w)}{w \cdot x} \leqq RM^{mult}(y,x)(\delta_1^* s_1 + \dots + \delta_N^* s_N) \tag{4}$$

where

$$\delta_i^* = \frac{\lambda_i^*}{\left(\prod_{n=1}^N \lambda_n^* \right)^{1/N}}, \quad i = 1, \dots, N \tag{5}$$

and $s_n = \frac{w_n x_n}{w \cdot x}$, $(n=1,\dots, N)$ are the factor shares and $\sum_{n=1}^N s_n = 1$. The expression $RM^{mult}(y, x)(\delta_1^* s_1 + \dots + \delta_N^* s_N)$ shows a technical efficiency component in comparison with the standard cost efficiency measure, $C(y,w)/wx$, because the new Eq. (4) is equivalent to $C(y,w) \leq w_1 \lambda_1^* x_1 + \dots + w_N \lambda_N^* x_N$. Incidentally, note that:

$$RM^{mult}(y,x) = 1 \quad \text{if and only if} \quad \lambda_1^* = \dots = \lambda_N^* = 1. \tag{6}$$

Furthermore, note that by closing the inequality with allocative efficiency, we have a cost decomposition relative to the efficient subset $EffL(y)$. Three comments are of particular consideration here:

i. $\delta_n^* = \frac{\lambda_n^*}{(\prod_{n=1}^N \lambda_n^*)^{1/N}}$ is the n-th input efficiency relative to the multiplicative Russell measure for $n = 1, \dots, N$.
ii. If $\lambda_n^* = \lambda^* \ \forall n = 1, \dots, N$, then the Farrell decomposition is obtained.
iii. The share s_n is the weight reflecting relative importance of n-th input in total cost.

[4] This section follows and expands on Färe, Grosskopf and Zelenyuk [19].