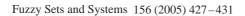


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## Remarks on basics of fuzzy sets and fuzzy multisets

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#### **Abstract**

Two remarks on basic aspects of fuzzy sets are given. They are notation for the  $\alpha$ -cuts and a commutative property in the extension principle. Application of the commutative property to fuzzy multisets is moreover considered. © 2005 Elsevier B.V. All rights reserved.

Keywords: Commutative properties; Extension principle; Fuzzy multiset

#### 1. Introduction

Two basic points on fuzzy sets are remarked in this small report; they are notation for  $\alpha$ -cuts and commutative properties between set operations, and the extension principle. We emphasize the usefulness of a commutative property between a set operation and an  $\alpha$ -cut.

Application of the commutativeness to fuzzy multiset image is also described where basic operations and a multiset image are defined.

#### 2. The $\alpha$ -cuts

Good notations will lead to deeper understanding of a theory. In the author's opinion there are rooms of improving notations in fuzzy sets. An example is the  $\alpha$ -cuts.

The strong  $\alpha$ -cut is the crisp set where the membership is 'greater than' the value of  $\alpha$  whereas the weak  $\alpha$ -cut is the crisp set where the membership is 'greater than or equal to' the value of  $\alpha$ .

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Assume that X is a metric space and membership function  $\mu_A(x)$  is continuous as many real examples suppose. It is now easy to see that the strong  $\alpha$ -cut gives an open set whereas the weak  $\alpha$ -cut provides a closed set. From this observation good notations for  $\alpha$ -cuts are  $]A[_{\alpha}$  for a strong  $\alpha$ -cut, and  $[A]_{\alpha}$  for an weak  $\alpha$ -cut.

It is well-known that  $\alpha$ -cuts commute with the basic set operations  $\cup$  and  $\cap$  such as  $]A \cup B[_{\alpha} = ]A[_{\alpha} \cup ]B[_{\alpha}, [A \cap B]_{\alpha} = [A]_{\alpha} \cap [B]_{\alpha}$ , etc. Such commutative properties are very important in understanding an old theory and in developing a new theory.

#### 3. Extension principle

Let Y be another set and  $f: X \to Y$ . Most literature describes the extension principle as  $\mu_{f(A)}(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$ , etc. The author does not oppose this definition, but there is more to be considered in relation to the crisp definition of the set image.

Let us consider a simple crisp example. We use a simple function  $\{\cdot\}: x \xrightarrow{\{\cdot\}} \{x\}$  that maps an element to a set that consists of that element alone. Then we have  $f(\{x\}) = \{f(x)\}$  from the standard crisp definition. Thus f and  $\{\cdot\}$  are commutative. Now, suppose  $B = \{x, x'\} = \{x\} \cup \{x'\}$ . We have  $f(B) = f(\{x\}) \cup \{x'\}) = \{f(x)\} \cup \{f(x')\}$ . Namely the map  $\xrightarrow{\{\cdot\}}$  and the operation  $\cup$  commute with  $f(\cdot)$ .

More generally, we note  $f(A) = \bigcup_{x \in A} \{f(x)\}$ . This equation is applicable to fuzzy set images.

We return to the first map  $x \mapsto \{x\}$  and generalizes it to fuzzy sets. For this purpose consider a fuzzy point  $(x, \mu_A(x))$  and  $(x, \mu_A(x)) \mapsto \{(x, \mu_A(x))\}$ . The last  $\{(x, \mu_A(x))\}$  is a fuzzy set which has nonzero membership at x alone. We can write  $A = \{(x, \mu_A(x)) \in X \times [0, 1]\} = \bigcup_{x \in X} \{(x, \mu_A(x))\}$ . We now have  $f(A) = \{(f(x), \mu_A(x)) \in Y \times [0, 1]\} = \bigcup_{x \in X} \{(f(x), \mu_A(x))\}$ . The last expression has redundancy when f(x) = f(x') for  $x \neq x'$ . Naturally the union operation is taken for the redundant case and we have the image of a fuzzy set. In other words, the rewriting rule  $\cdot \mapsto f(\cdot)$  is used in common:  $A = \bigcup_{x \in A} \{x\} \Rightarrow f(A) = \bigcup_{x \in A} \{f(x)\}$  for a crisp set, and  $A = \bigcup_{x \in X} \{(x, \mu_A(x))\} \Rightarrow f(A) = \bigcup_{x \in X} \{(f(x), \mu_A(x))\}$  for a fuzzy set. To note the commutative property thus appears useful.

We now have a new problem, however. The above definition using fuzzy points is not equivalent to the extension principle. To see this, notice that  $[f(A)]_{\alpha} \neq f([A]_{\alpha})$  in general: the weak  $\alpha$ -cut and the functional image is not commutative. In contrast, it is easily seen that  $[f(A)]_{\alpha} = f([A]_{\alpha})$  from the above definition, since  $y \in [f(A)]_{\alpha}$  means there exists an x such that y = f(x) and  $x \in [A]_{\alpha}$ , which does not hold from the extension principle.

Why does such a difference occur? The reason is that infinite union of fuzzy points may lead to a 'half-open' fuzzy point. For example,  $\bigcup_{1 \leq n < \infty} (x, [0, 1 - 1/n]) = (x, [0, 1])$ . Thus the infinite union fails to have a membership. On the other hand, if we use an identity map I(x) = x, the extension principle leads to  $I(\bigcup_{1 \leq n < \infty} (x, [0, 1 - 1/n])) = (x, [0, 1])$ .

Such a difficulty has been known by theoretical researchers, and we may have different attitudes.

- (i) Satisfy with finite operations. Then the commutative property and the extension principle are consistent.
- (ii) Give up the commutative property; it is enough to have the extension principle.
- (iii) Analyse the cases when the commutative property holds or not, and derive conditions for the commutativeness.

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