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Interference of probabilities in the classical probabilistic framework

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Abstract

The notion of context (complex of physical conditions) is basic in this paper. We show that the main structures of quantum theory (interference of probabilities, Born's rule, complex probabilistic amplitudes, Hilbert state space, representation of observables by operators) are present in a latent form in the classical Kolmogorov probability model. However, this model should be considered as a calculus of contextual probabilities. In our approach, it is forbidden to consider abstract context-independent probabilities: "first context and then probability." We start with the conventional formula of total probability for contextual (conditional) probabilities and then we rewrite it by eliminating combinations of incompatible contexts from consideration. In this way we obtain interference of probabilities without to appeal to the Hilbert space formalism or wave mechanics. Our contextual approach is important for demystification of quantum probabilistic formalism. This approach gives the possibility to apply quantum-like models in domains of science different from quantum theories, e.g., in economics, finance, social sciences, cognitive sciences, psychology.

Keywords: Kolmogorov model; Interference; Complex amplitude; Context; Conditional probability; Quantum physics; Economics; Finance; Social sciences; Cognitive sciences; Psychology

1. Introduction

In this article we present a so-called contextual viewpoint of the origin of quantum (conditional) probabilities, see [4–9] for details. Such an approach gives the possibility to unify classical Kolmogorov

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(measure theoretical) and quantum (Hilbert space) probability theories by constructing a natural representation of the Kolmogorov model in a complex Hilbert space. Thus, in the contextual approach quantum probabilistic behavior (in particular, *interference of probabilities*) is simply a consequence of a very special representation of Kolmogorov probabilities—by complex amplitudes (vectors in a complex Hilbert space). Each representation is based on a fixed pair of observables (Kolmogorov random variables) a and b, *reference observables*, which produce the contextual image of a Kolmogorov probability space in a complex Hilbert space.

In articles [4–9], the Hilbert space representation of the Kolmogorov model was constructed on the basis of dichotomous reference observables. This is a rather strong restriction, especially in the light of the Kochen–Specker "no-go" theorem which is valid only for Hilbert spaces of dimension > 2, see, e.g., [3] for details. In this paper we construct a quantum-like representation of the Kolmogorov model for arbitrary reference observables yielding a finite number of values. This is a quite complicated construction based on an inductional representation of conditional probabilities by complex amplitudes (in such a way that Born's rule holds).

Our contextual approach is important for demystification of quantum probabilistic formalism. This approach gives the possibility to apply quantum-like models in domains of science different from quantum theories, e.g., in economics, finance, social sciences, cognitive sciences, psychology, see [1,2,9,10].

2. Contextual viewpoint to the Kolmogorov model and interference of probabilities

Let $\mathbf{K} = (\Omega, \mathbf{F}, \mathbf{P})$ be a Kolmogorov probability space. By the standard Kolmogorov axiomatics sets $A \in \mathbf{F}$ represent *events*. In our simplest model of *contextual probability* (which can be called the Kolmogorov contextual space) the same system of sets, $\mathbf{F}$, is used to represent complexes of experimental physical conditions—*contexts.* [1]

The conditional probability is mathematically defined by Bayes' formula

$$\mathbf{P}(A/C) = \mathbf{P}(AC)/\mathbf{P}(C), \quad \mathbf{P}(C) \neq 0.$$

In our contextual model this probability has the meaning of the probability of occurrence of the event A under the complex of physical conditions C. [2]

Let $\mathbf{A} = \{A_n\}$ be finite or countable *complete group of disjoint contexts* (or in the event-terminology—complete group of disjoint events)

$$A_i A_j = \emptyset, i \neq j, \quad \cup_i A_i = \Omega.$$

[1] Thus, depending on circumstances a set $O \in \mathbf{F}$ will be interpreted either as event or as context. We shall sharply distinguish events and contexts on phenomenological level, but we shall use the same mathematical object $\mathbf{F}$ to represent both events and contexts in a mathematical model. In principle, in a mathematical model events and contexts can be represented by different families of sets, e.g., in Renye's model. We will not do this from the beginning. But, later we will fix a proper subfamily of contexts $\mathbf{CONT} \subset \mathbf{F}$.

[2] Thus, it would be more natural to call $\mathbf{P}(A/C)$ a *contextual probability* and not *conditional probability*. Roughly speaking to find $\mathbf{P}(A/C)$ we should find parameters ω^A favoring for the occurrence of the event A among parameters ω^C describing the complex of physical conditions C.