Available online at www.sciencedirect.com
SCIENCE DIRECT®

Fuzzy Sets and Systems 155 (2005) 150–163

FUZZY sets and systems

www.elsevier.com/locate/fss

On central algorithms of approximation under fuzzy information

Svetlana V. Asmuss*, Alexander P. Šostak

Department of Mathematics, University of Latvia, 19 Rainis Boul., Riga LV-1586, Latvia

Available online 9 June 2005

Abstract

We consider the problem of approximation of an operator by information described by n real characteristics in the case when this information is fuzzy. We develop the well-known idea of an optimal error method of approximation for this case. It is a method whose error is the infimum of the errors of all methods for a given problem characterized by fuzzy numbers in this case. We generalize the concept of central algorithms, which are always optimal error algorithms and in the crisp case are useful both in practice and in theory. In order to do this we define the centre of an L-fuzzy subset of a normed space. The introduced concepts allow us to describe optimal methods of approximation for linear problems using balanced fuzzy information.

MSC: 04A72; 41A65

Keywords: L-fuzzy number; Fuzzy information; Central algorithm of approximation

1. Introduction

We consider the problem of approximation of an operator B defined on a set X and taking values in a normed space Y. To find an approximation of the exact value Bx, we must know something about the problem element $x \in X$. We denote the information of x by Ax and suppose that this information is given by an information vector z (information vectors z) with n real coordinates. Usually the information is imprecise; thus there may exist many different information vectors z, corresponding to one problem element $x \in X$. Under such an interpretation it seems natural to consider information Ax as fuzzy, rather than crisp, i.e. to realize it as an L-fuzzy subset of $\mathbb{R}^n$ characterized by a function $Ax : \mathbb{R}^n \to L$, where

* Corresponding author. Tel.: +371 7033726; fax: +371 7033701.
E-mail address: asmuss@lanet.lv (S.V. Asmuss).

doi:10.1016/j.fss.2005.05.018

the value $Ax(z)$ in a lattice L describes the belongness degree of the information vector $z \in \mathbb{R}^n$ to the set Ax. Thus we set that the information is given by an operator $A : X \to L^{\mathbb{R}^n}$ called an information operator. We denote by (A, B) the problem of approximation of an operator B under the information given by an operator A.

By a method (an algorithm) of solution of the problem (A, B) we mean that any operator $\varphi : \mathbb{R}^n \to Y$. For an information vector $z \in \mathbb{R}^n$ the value $\varphi(z)$ must approximate the exact value of solution Bx for the problem element x corresponding to the information vector z:

$$x \in X \Longrightarrow z \tilde{\in} Ax \in L^{\mathbb{R}^n} \Longrightarrow \varphi(z) \approx Bx \in Y.$$

For most problems the information operator is not one-to-one and a given information vector z does not determine the problem element x (the solution element Bx) in a unique way. Thus, there may exist many different problem elements (solution elements) with the same information. Such problems also cannot be solved exactly. For problems which can be solved only approximately the notion of the error of a method of approximation plays a fundamental role.

The principal aim of this paper is to generalize for the case of fuzzy information the idea of an optimal error method of approximation (the method whose error is the infimum of the errors of all methods for a given problem using information) and the concept of a central algorithm (which is always an optimal error algorithm and in the crisp case is useful in practice as well as in the general theory). Notice that the error of a method in this case is characterized by an L-fuzzy real number which is obtained as the supremum of a certain L-fuzzy set of real numbers. The problem of defining the supremum and the infimum of L-fuzzy set of real numbers as well as the supremum and the infimum of a crisp set of L-fuzzy numbers was considered in our papers [1,2] devoted to a fuzzy approach to the problems of approximation theory. The main results of this paper without proofs are given in [3].

2. Preliminaries

2.1. Some concepts and results from the lattice theory

In questions where lattices are involved, our main source of references is [8]. However, for the reader's convenience, here we reproduce some definitions, notation and results which will be needed in the sequel.

Let $L = (L, \leqslant, \wedge, \vee)$ be a complete lattice.

In particular, $\mathbf{2} = \{0, 1\}$ is the two-point lattice and $\mathbb{I} = [0, 1]$ is the closed unit interval equipped with the natural less-or-equal relation. If $A \subset L$ then we denote $\bigvee A := \bigvee\{\alpha \mid \alpha \in A\}$ and $\bigwedge A := \bigwedge\{\alpha \mid \alpha \in A\}$. In particular, $\bigwedge L := 0$ and $\bigvee L := 1$ are universal lower and upper bounds of L, respectively.

We write $\alpha < \beta$ if $\alpha \leqslant \beta$ and $\alpha \neq \beta$.

On each complete lattice one can define a new transitive relation $\ll$ (the so-called "way below relation") as follows:

$$\alpha \ll \beta \quad \text{whenever for every } A \subset L \text{ with } \beta \leqslant \bigvee A$$
$$\text{there exists a finite subset } B \subset A \text{ such that } \alpha \leqslant \bigvee B.$$

Equivalently,

$$\alpha \ll \beta \quad \text{iff for every directed } D \subset L \text{ s.t. } \beta \leqslant \bigvee D \text{ there exists } d \in D \text{ s.t. } \alpha \leqslant d.$$