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Random intervals as a model for imprecise information☆

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Abstract

Random intervals constitute one of the classes of random sets with a greater number of applications. In this paper, we regard them as the imprecise observation of a random variable, and study how to model the information about the probability distribution of this random variable. Two possible models are the probability distributions of the measurable selections and those bounded by the upper probability. We prove that, under some hypotheses, the closures of these two sets in the topology of the weak convergence coincide, improving results from the literature. Moreover, we provide examples showing that the two models are not equivalent in general, and give sufficient conditions for the equality between them. Finally, we comment on the relationship between random intervals and fuzzy numbers.

Keywords: Random sets; Random intervals; Measurable selections; Dempster–Shafer upper and lower probabilities; Fuzzy numbers; Weak convergence

1. Introduction

Random set theory has been applied in such different fields as economy [20], stochastic geometry [27] or when dealing with imprecise information [26]. Within random sets, random intervals are especially interesting, as the works carried out in [9,11,24] show. One of their advantages respect to other types of

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random sets is their easy interpretation as a model for uncertainty and imprecision. Consider a probability space $(\Omega, \mathcal{A}, P)$ and a random variable $U_0\colon\ \Omega \to \mathbb{R}$ modeling some behavior of the elements of Ω. Due to some imprecision in the observation of the values $U_0(\omega)$, or to the existence of missing data, we may not precisely know the images of the elements from Ω by U_0. A possible model for this situation would be to give, for any ω in Ω, upper and lower bounds of its image by U_0 (i.e., a margin for error in the observation); we obtain then an interval $\Gamma(\omega) = [A(\omega), B(\omega)]$ which we assume is certain to include the value $U_0(\omega)$.

Given this model, we can study which is the information conveyed by the multi-valued mapping Γ about the probability distribution of the random variable U_0. On the one hand, we know that U_0 belongs to the class of the random variables whose values are included in the images of the multi-valued mapping. Thus, its probability distribution belongs to the class of the probability distributions of these random variables. We will denote this class by $P(\Gamma)$. On the other hand, the probability induced by U_0 is bounded between the *upper and lower probabilities* of Γ. These two functions were introduced by Dempster in 1967 [8]; see also the previous work by Strassen in [36]. We will denote them by P^* and P_*, respectively, and will denote the class of the probabilities bounded between them by $M(P^*)$. Hence, we can consider two models of the available information: the class of probability distributions of the *measurable selections* of the random set, $P(\Gamma)$, and the set of probabilities bounded between the upper and the lower probability of the random set, $M(P^*)$. The first of these two models is the most precise we can consider with the available information, so $P(\Gamma) \subseteq M(P^*)$; however, the class $M(P^*)$ is more interesting from an operational point of view, because it is convex, closed in some cases, and is uniquely determined by the values of P^*. The goal of this paper is to study the relationship between these two models.

The paper is organized as follows: in Section 2, we introduce some concepts and notations that we will use in the rest of the paper. In Section 3, we recall some useful results from the literature and study the relationship between the classes $P(\Gamma)$ and $M(P^*)$. In Section 4, we establish sufficient conditions for the equality between these two sets of probabilities, first for random closed intervals and later for random open intervals. Section 5 contains some comments on the connection between random intervals and fuzzy numbers. Finally, in Section 6 we give our conclusions and open problems on the subject.

2. Preliminary concepts

Let us introduce the notation we will use throughout the paper. We will denote a probability space by $(\Omega, \mathcal{A}, P)$, a measurable space $(X, \mathcal{A}')$ and a multi-valued mapping, $\Gamma : \Omega \to \mathcal{P}(X)$. $\mathcal{N}_P$ will denote the class of null sets with respect to a probability *P*, and δ_x will denote the degenerate probability distribution on a point *x*. Given a topological space (X, τ), β_X will denote its Borel σ-field, that is, the σ-field generated by the open sets. In particular, $\beta_{\mathbb{R}}$ will denote the Borel σ-field on $\mathbb{R}$, and given $A \in \beta_{\mathbb{R}}$, β_A will denote the relative σ-field on *A*. On the other hand, λ will denote the Lebesgue measure on $\beta_{\mathbb{R}}$, and λ_A will denote the restriction of λ to β_A. Given a random variable $U\colon\Omega \to \mathbb{R}$, $F_U : \mathbb{R} \to [0, 1]$ will denote its distribution function, and $P_U\colon\beta_{\mathbb{R}} \to [0, 1]$, its induced probability. A set of probabilities will be called $\mathcal{W}$-compact (resp. $\mathcal{W}$-closed) when it is compact (resp., closed) in the topology of the weak convergence. A multi-valued mapping will be called compact (resp., closed, open) when $\Gamma(\omega)$ is a compact (resp., closed, open) subset of *X* for every $\omega \in \Omega$. Most of the multi-valued mappings to appear in this paper will take values on $(\mathbb{R}, \beta_{\mathbb{R}})$; nevertheless, we will also consider the case where the final space is Polish.