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Extensions of belief functions and possibility distributions by using the imprecise Dirichlet model

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Abstract

A belief function can be viewed as a generalized probability function and the belief and plausibility measures can be regarded as lower and upper bounds for the probability of an event. However, the classical probabilistic interpretation used for computing belief and plausibility measures may be unreasonable in many real applications when the number of observations or measurements is rather small. In order to overcome this difficulty, Walley's imprecise Dirichlet model is used to extend the belief, plausibility and possibility measures. An interesting relationship between belief measures and sets of multinomial models is established. Combination rules taking into account reliability of sources of data are studied. Various numerical examples illustrate the proposed extension.

Keywords: Approximate reasoning; Belief measures; Dirichlet distribution; Multinomial model; Possibility theory; Dempster–Shafer theory; Imprecise probability theory

1. Introduction

Evidence theory or Dempster–Shafer theory [9,21] provides us with an appropriate mathematical model of uncertainty when information is not complete or when the result of each observation is not point-valued but set-valued, so that it is not possible to assume the existence of a unique probability measure. From this point of view, Dempster–Shafer theory can be interpreted as a generalization of probability theory where probabilities are assigned to sets as opposed to mutually exclusive singletons. There are three main functions in Dempster–Shafer theory: the basic probability assignment function, the belief function, and

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the plausibility function. As indicated in [13,20], generally, the term “basic probability assignment” does not refer to probability in the classical sense, but it is useful to interpret the basic probability assignment as a classical probability, and the framework of Dempster–Shafer theory can support this interpretation. Under this circumstance, Dempster–Shafer theory allows one to calculate upper (plausibility) and lower (belief) bounds for a probability of occurrence of an outcome, and many real applications use this interpretation.

At the same time, real applications meet some difficulties by using the classical probabilistic interpretation because they require a large number of observations of events. If this number is small, inferences become too precise and incautious. Therefore, an approach to extend the belief and plausibility functions in order to take into account a lack of sufficient statistical data is proposed in the paper. This approach is based on using Walley's imprecise Dirichlet model [26]. Since the possibility distribution function [10] can be regarded as a special case of belief and plausibility functions, the same extension is applied to the possibility measure.

The paper is organized as follows. Basic formal definitions of Dempster–Shafer theory are given in Section 2. The imprecise Dirichlet model is considered in Section 3. A relationship between a set of multinomial models and statistical data in the form of subsets of a universal set is established in Section 4. The extended belief and plausibility functions are proposed in the same section. The extended belief and plausibility functions are explained in terms of a multivalued sampling process in Section 5. Some important properties of the obtained extensions are investigated in Section 6. The extended possibility distribution is described in Section 7. Two combination rules are studied in Section 8. Lower and upper probability distributions and expectations produced by the extended belief and plausibility functions are considered in Section 9.

2. Belief functions

Let U be a universal set under interest, usually referred to in evidence theory as the *frame of discernment*. Suppose N observations were made of an element $u \in U$, each of which resulted in an imprecise (non-specific) measurement given by a set A of values. Let c_i denote the number of occurrences of the set $A_i \subseteq U$, and $\mathcal{P}(U)$ the set of all subsets of U (power set of U). A frequency function m, called *basic probability assignment*, can be defined such that [9,16,21]:

$$m : \mathcal{P}(U) \to [0, 1],$$

$$m(\varnothing) = 1, \qquad \sum_{A \in \mathcal{P}(U)} m(A) = 1.$$

Note that the domain of basic probability assignment, $\mathcal{P}(U)$, is different from the domain of a probability density function, which is U. According to [9], this function can be obtained as follows:

$$m(A_i) = c_i / N. \tag{1}$$

If $m(A_i) > 0$, i.e. A_i has occurred at least once, then A_i is called a *focal element*.

According to [21], the *belief* $Bel(A)$ and *plausibility* $Pl(A)$ measures of an event $A \subseteq \Omega$ can be defined as

$$Bel(A) = \sum_{A_i : A_i \subseteq A} m(A_i), \quad Pl(A) = \sum_{A_i : A_i \cap A \neq \varnothing} m(A_i). \tag{2}$$