Omega 63 (2016) 123–133

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Late season low inventory assortment effects in the newsvendor problem ☆

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ARTICLE INFO

Article history:
Received 16 January 2015
Accepted 9 October 2015
Available online 19 October 2015

Keywords:
Inventory control
Marketing

ABSTRACT

The assumption of the newsvendor being able to satisfy demand as long as on-hand inventory is positive does not hold for a non-homogenous product. Consumers who do not find a unit of the product which satisfies their secondary features preferences may not purchase the product even though the newsvendor has positive on-hand inventory. This is likely to occur late in the season as inventory level declines. We solve a newsvendor problem in which the probability of purchase by consumers is increasing in on-hand inventory for any inventory level below that which is needed to have a complete assortment. We identify the sufficient optimality condition for the order quantity. We show that, unlike the case of inventory-dependent demand models in the literature, the optimal order quantity may decrease due to the assortment effect. We investigate two types of pre-end of season discounts, immediate all-units and delayed, as ways to mitigate the late season assortment effect and show that in some cases, they can increase the newsvendor's profit and free up the shelf space for other products.

1. Introduction

The classical newsvendor problem is to find a perishable product's order quantity that maximizes the expected profit under probabilistic demand [14]. If the newsvendor's inventory falls to zero then incoming demand goes unsatisfied. The assumption that the newsvendor can satisfy demand as long as on-hand inventory is positive does not hold for cases of heterogenous products such as fashion and food. For example, winter jackets that are ordered by a retailer in a complete assortment which includes six sizes, each in four colors, results in the complete assortment having 24 different size–color combinations. The retailer's order may include the same number of each size–color combination. For example, the retailer may order 240 jackets, i.e. 10 jackets in each color–size combination. Alternatively, the retailer may order a larger number of more popular sizes and smaller number of less popular sizes in the total 240 jackets order. In the early part of the selling season, there will be some units on-hand of every color–size combination. As sales continue and inventory level declines, it will be less likely to have a complete assortment, i.e. every color–size combination, in stock. When the inventory level reaches 23, it is impossible to have a complete assortment. Obviously, as some size–color combinations are no longer on-hand, an arriving consumer has a lower probability of finding a unit satisfying her/his secondary features preferences, such as color or exact size, and is less likely to purchase the product. We refer to this inability to fulfill consumers' secondary features preferences as the *assortment effect*. The assortment here refers to a product variations, e.g. a specific design winter jacket or real Christmas trees, and not within a product category, which is analyzed in multi-product newsvendor models with product substitution [9].

Examples of the assortment effect in the newsvendor are common. A real Christmas tree newsvendor is very unlikely to sell the last few trees as consumers do not even come in when the newsvendor has only a few trees left and when they do come in, it is unlikely that they will find a tree they like. This assortment effect is also observed for fresh produce, fresh meats, and clothing.

In this paper, we examine how the low inventory assortment effect influences the optimal order quantity and the expected profit of the newsvendor. We also examine the use of an immediate all-units pre-end of season discount, i.e. the discount is implemented as soon as inventory falls below the complete assortment level, due to high shelf space cost [3]. We also examine the case in which the retailer follows optimal discounting, i.e. may delay the discount and sell some units at the regular price while inventory is below the complete assortment level before using a pre-end of season discount.

The remainder of this paper is organized as follows. In Section 2, we summarize the related literature. In Section 3, we quantify the impact of the assortment effect on sales and derive the optimal order quantity when the effect is present. In Section 4, we examine the use of an all-units pre-end of season discount offered as soon

☆This manuscript was processed by Associate Editor Leung.

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http://dx.doi.org/10.1016/j.omega.2015.10.008
0305-0483/

as the inventory level falls short of a complete assortment to free up the shelf space. In Section 5, we allow the discounting to be delayed, which may be optimal when shelf-space is not very costly, and identify the optimal inventory level at which to offer the pre-end of season discount. In Section 6, we present a numerical analysis and some insights. We present the conclusion and suggest directions for future research in Section 7.

2. Literature review

Our model is related to models in four areas, inventory-dependent demand, assortment management, clearance pricing, and the newsvendor model. The role of inventory in influencing demand has been examined in both deterministic and stochastic inventory-dependent demand models. In these models, demand is assumed to be an increasing function of the inventory level. The increase in demand is attributed to the "advertising effect" in which a large shelf space and display quantity signal product popularity to consumers [17]. The "advertising effect" depends on the quantity of displayed product, which may or may not be homogenous. For a comprehensive review of this literature the reader is referred to Urban [22].

Inventory may also influence demand through the "selective effect" in which more units of a product which are not identical, e.g. fresh produce, real Christmas trees, fashion, provide consumers with more choices and induce them to purchase [24]. The "assortment effect" analyzed in this paper plays a similar role as the selective effect but in the direction of decreasing sales, i.e. if a high inventory level of a heterogenous product increases demand, then a low inventory level below complete assortment may decrease sales. When a product's inventory level is low, some consumers whose reservation prices would have been met if they found the right product, i.e. one matching their secondary features preferences, will not purchase the product because the remaining selection lacks these features, e.g. color, size, freshness [19]. In this case, sales will be lower than the actual demand in spite of the fact that the newsvendor does not stock out of the product.

Assortment can be measured in two ways, breadth and depth [16]. Breadth is measured by the number of different product categories a retailer carries. Depth is measured by how many stock keeping units (SKUs) a retailer carries in each category. In this sense, depth includes both variants (SKUs) of the product, e.g. color and size, a retailer carries and the inventory level for each SKU. Finding the optimal breadth of the assortment and its pricing over time has received considerable attention in the literature [13,15,18].

The optimal depth and pricing of the assortment has received less attention than breadth. Caro and Gallien [4] analyzed a problem in the fast-fashion industry where stores of a retail chain remove products from display whenever one of its key sizes stocks out and optimized the quantities to ship to stores to maximize profits. Caro and Gallien [5] analyzed clearance pricing of a global retail chain in which some clearance inventory is moved between stores. Each product group, such as "woman blazers," are broken into clusters and different pricing policies are applied to the clusters. Smith and Achabal [21] examined clearance pricing and inventory policies for retail chains. They argued that while the sales rate is not affected by high inventory levels, the sales rate is decreased by low inventory levels, in particular for clothing when there is an incomplete selection of sizes and colors. They refer to an assortment of remaining inventory without complete selection of sizes and colors as "broken assortment." Their model focused on the optimal price trajectory for the product. Abbott and Palekar [1] defined the minimum presentation quantity as the minimum shelf space allocated for a product to generate sales corresponding to inherent demand and identified the optimal replenishment policy for a nonperishable product. Kalyanam et al. [12] analyzed how the presence of every color and size, which they refer to as product attributes, affect sales. They found that substitution in case of shortages of some items is not common and was limited to a few colors. Furthermore, there was little substitution across sizes. This finding supports the conclusion that consumers who cannot find a unit with their preferred secondary features get less utility from purchasing one with different secondary features, e.g. color or size. Vakhutinsky et al. [23] developed a markdown optimization (MDO) model in which demand is a function of price, seasonality, and inventory. The inventory effect is captured using power of the ratio of on-hand inventory to a critical inventory level. They then obtained closed-form solution for price in continuous time and developed a solution algorithm to a dynamic programming formulation for the discrete time case.

There are some models in the literature which analyze pre-end of season (i.e. in-season) price adjustments for the newsvendor. Feng and Xiao [7] identified the optimal times to switch between predetermined prices based on the time remaining in the season and inventory. They assumed demand follows a price-dependent poisson arrival process. Chung et al. [6] allowed an in-season price change which is implemented after the demand forecast is updated based on the realized demand up to the price change time. Forghani et al. [8] assumed the demand rate is a random normal variable whose probability density function (pdf) is known and whose parameters depend on the selling price according to one of three functional forms. The authors allowed one in-season price adjustment. While our model also allows an in-season price adjustment, the purpose of the adjustment is to mitigate the assortment effect and the adjustment does not affect sales until the remaining inventory falls below the level needed for complete assortment, if it does. In this respect, our model can be used with any of the above models to make a second in-season price adjustment when or after the inventory level needed for complete assortment is reached.

The above late season assortment effect is also related to shelf space management. When the inventory level of a product falls below the complete assortment level, the newsvendor can keep the shelf space allocation to the product unchanged and try to sell the remaining units. However, when the shelf space cost is large [3], the newsvendor may decide to reallocate the shelf space of the product as soon as the complete assortment level is reached or at some level below it. The remaining units of the product are then grouped with other products whose remaining on-hand inventories are also less than their complete assortment levels in a discount area. For example, fashion retailers have racks displaying the "sale" or "clearance" sign where these products are compactly grouped. Some retailers have a minimum on-hand inventory for each product called "fixture fill" which is the quantity required for adequate presentation [21], which may be considered as the complete assortment level or a fraction of it.

Our model assumes that the regular price is exogenous and therefore, unlike price-dependent demand newsvendor models [11,20], sales depend only on the availability of a complete assortment. Our model contributes to the literature by examining the effect of a broken assortment on the optimal order quantity. While most models examine pricing when a broken assortment occurs, we identify the optimal order quantity in the presence of this assortment effect. We show that the newsvendor may respond to the assortment effect by increasing the order quantity. However, there are cases in which the newsvendor responds by decreasing the order quantity. We also analyze two policies the newsvendor can use in response to the assortment effect. In the first, the newsvendor discounts the product as soon as a broken assortment occurs, i.e. all-units discount. We find that if this discount is used, then the newsvendor is more likely to increase the order quantity. If all consumers are aware of the discount, then the discount may significantly increase the newsvendor's profit. We also identify the