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Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis in multiple criteria hierarchy process for the Choquet integral preference model ☆

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ABSTRACT

The paper deals with two important issues of Multiple Criteria Decision Aiding: interaction between criteria and hierarchical structure of criteria. To handle interactions, we apply the Choquet integral as a preference model, and to handle the hierarchy of criteria, we apply the recently proposed methodology called Multiple Criteria Hierarchy Process. In addition to dealing with the above issues, we suppose that the preference information provided by the Decision Maker is indirect and has the form of pairwise comparisons of criteria with respect to their importance and pairwise preference comparisons of some pairs of alternatives with respect to some criteria. In consequence, many instances of the Choquet integral are usually compatible with this preference information. These instances are identified and exploited by Robust Ordinal Regression and Stochastic Multiobjective Acceptability Analysis. To illustrate the whole approach, we show its application to a real world decision problem concerning the ranking of universities for a hypothetical Decision Maker.

1. Introduction

Multiple Criteria Decision Aiding (MCDA) helps Decision Makers in solving choice, ranking and sorting problems concerning a set of alternatives evaluated on multiple criteria (see [15] for a collection of state-of-the-art surveys on MCDA). Taking into account the preferences of a particular Decision Maker (DM), in choice problems, a subset of best alternatives has to be chosen; in ranking problems, alternatives have to be partially or totally rank ordered from the best to the worst, while in sorting problems each alternative has to be assigned to one or more contiguous preferentially ordered classes. In order to deal with any of these problems, the evaluations of the alternatives on the considered criteria have to be aggregated by a preference model, which can be either a value function [33], or an outranking relation [8,17], or a set of decision rules [29,43].

Nowadays, MCDA is facing three important methodological challenges: handling a complex structure of criteria, dealing with interactions between criteria, and reducing the cognitive effort of the DMs in interaction with MCDA methods. These challenges are usually handled separately, however, they often concern the same decision problem.

In particular, with respect to the complex structure of criteria having the form of a hierarchy, the Analytic Hierarchy Process (AHP) [41], and then the Multiple Criteria Hierarchy Process (MCHP) [13] have been proposed. While AHP requires preference information at all levels of the hierarchy in the form of exhaustive pairwise comparisons, and provides recommendations at the comprehensive level only, MCHP accepts a partial preference information in form of pairwise comparisons of some alternatives at some levels of the hierarchy, and provides recommendations at all levels.

As to the challenge of interaction, it is present when evaluation criteria are not mutually preferentially independent [33]. To deal with interactions, MCDA methods use non-additive integrals, such as the Choquet integral (see [9] for the Choquet integral definition, and [24] for the application of non additive integrals in MCDA), the Sugeno integral [46], and some of their generalizations [26,28,32,36]. The preferential independence condition has also

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been smoothed in multiplicative and multilinear utility functions [33], but due to the high number of parameters that have to be elicited from the DM, their use has not been very successful in real world applications [45].

Moreover, the interaction between criteria has been recently considered in the ELECTRE methods [16] and in PROMETHEE methods [10]. It was also handled in artificial intelligence approaches, by weakening the preference independence condition in GAI-networks [23], as well as UCP-networks [7]. They are based on the concept of Generalized Additive Independence (GAI) decomposition introduced by Fishburn [18], which permits to aggregate performances on considered criteria through the sum of marginal utilities related to subsets of criteria. Yet another approach, recently proposed to deal with the interaction between criteria [31] is based on an enriched additive value function that is composed of the usual sum of marginal value functions related to each one of considered criteria and some additional terms expressing a bonus (in the case of positive interaction) or a penalty (in the case of negative interaction), incurred for interaction between some criteria. In this approach, the pairs of criteria for which there exists a positive or negative interaction are inferred through ordinal regression on the basis of preference information given by the DM on some reference alternatives.

The aforementioned aspects of hierarchy and interaction of criteria have been jointly analyzed and described in the hierarchical Choquet integral preference model [5]. Other studies devoted to modeling the hierarchy of criteria within the Choquet integral preference model can be found in [19–22,39,40,47]. Let us remark that their multi-step Choquet integral is different from our approach, since it requires the definition of a capacity at each node of the hierarchy of criteria. Consequently, their method considers Choquet integrals resulting from the aggregation of Choquet integrals at the subsequent level of the hierarchy, which is not the case of our approach.

As to the challenge of reducing the cognitive effort of the DM, one can observe the trend of abandoning direct elicitation of preference model parameters in favor of an indirect elicitation of preferences. In the direct elicitation, the DM is expected to provide values of all parameters of the considered preference model, while in the indirect elicitation, the DM is expected to provide preference information in the form of pairwise comparisons between some alternatives or criteria. There are known two MCDA methodologies based on the indirect elicitation of preferences, which explore the whole set of preference model parameters compatible with the preference information provided by the DM. These are the Robust Ordinal Regression (ROR) (see [30] for the paper introducing ROR, and [11,12] for surveys) and the Stochastic Multiobjective Acceptability Analysis (SMAA) (see [34] for the paper introducing SMAA, and [48] for a survey).

In this paper, we undertake all these three challenges together, combining the use of MCHP with the Choquet integral preference model on one hand and application of ROR and SMAA on the other hand. This combination is not straightforward, however, because it does not consist in chaining these three methods as they are, but in joint application of all of them, which needs some non-trivial adaptations. In this way, we extend the study presented in [5] by considering two new aspects:

- application of ROR to identify all instances of the Choquet integral preference model being compatible with the preference information provided by the DM; due to hierarchical structure of criteria, the DM can express preference information at a particular level of the hierarchical decomposition of the problem; in exchange, ROR provides robust recommendation in terms of necessary and possible preference relations at all levels of the hierarchy of criteria;
- application of SMAA to compute the frequency with which an alternative gets a particular position in the recommended ranking or the frequency with which an alternative is preferred to another one, at all levels of the hierarchy of criteria.

Let us observe that the methodology presented in this paper is not just a simple sum of the aforementioned three approaches, because MCHP requires that the Choquet integral preference model, SMAA and ROR are applied in all nodes of the hierarchy of criteria in a different way than in the case of a flat structure of criteria; the hierarchy requires a coordination of calculations in particular nodes, and moreover, the preference information does not need to be given in all nodes. Moreover, the approach is really adaptive with respect to the complexity of the decision problem considered, since on one hand, it permits decomposition of complex problems due to hierarchical structure of criteria and, on the other hand, it permits to adapt the Choquet integral from 1-additive form (linear) to k-additive form, depending on the preference information provided by the DM. Another aspect that we would like to underline here and that will be clear in the next sections is that the extension of the MCHP to the Choquet integral preference model does not require more parameters than the application of the Choquet integral preference model in the case of a flat structure of criteria. Indeed, the application of the Choquet integral in the case of criteria structured in a hierarchical way requires only the definition of a capacity on the set of elementary criteria and not of a capacity on each node of the hierarchy. Indeed, the capacities on the different nodes of the hierarchy can be easily obtained by the capacity defined on the elementary criteria only.

The highlights characterizing the approach presented in this paper are summarized briefly in the following paragraphs.

At the input, the DM is asked to provide the following preference information:

- comparisons related to importance and interaction of macro-criteria as well as between some elementary criteria, not necessarily belonging to the same macro-criterion;
- preference comparisons between alternatives at a comprehensive level as well as considering only a macro-criterion and, therefore, a particular aspect of the problem at hand.

At the output, the DM gets the following results again with respect to each node of the hierarchy as well as at a comprehensive level:

- necessary and possible preference relations resulting from NAROR;
- all the probabilistic indices supplied by SMAA applied to the k-additive Choquet integral preference model;
- the rankings of the alternatives, by applying the Choquet integral preference model assuming the barycenter of the capacities compatible with the preference information provided by the DM.

The paper is organized as follows. In Section 2, we introduce some basic concepts relative to the Choquet integral preference model, MCHP, hierarchical Choquet integral preference model, ROR and SMAA. In Section 3, the proposed methodology, combining SMAA and ROR applied to the hierarchical Choquet integral preference model, is presented. A real world multicriteria problem, related to the ranking of universities, illustrates the considered methodology in Section 4. Conclusions are drawn and some future directions of research are provided in Section 5.