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Review

Performance analysis of time-dependent queueing systems: Survey and classification ☆

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ABSTRACT

Many queueing systems are subject to time-dependent changes in system parameters, such as the arrival rate or number of servers. Examples include time-dependent call volumes and agents at inbound call centers, time-varying air traffic at airports, time-dependent truck arrival rates at seaports, and cyclic message volumes in computer systems.

There are several approaches for the performance analysis of queueing systems with deterministic parameter changes over time. In this survey, we develop a classification scheme that groups these approaches according to their underlying key ideas into (i) numerical and analytical solutions, (ii) approaches based on models with piecewise constant parameters, and (iii) approaches based on modified system characteristics. Additionally, we identify links between the different approaches and provide a survey of applications that are categorized into service, road and air traffic, and IT systems.

Contents

1. Introduction

Many queueing systems feature time-dependent changes in parameters. Examples of non-stationary parameters, such as the arrival rate or number of servers, include time-dependent call

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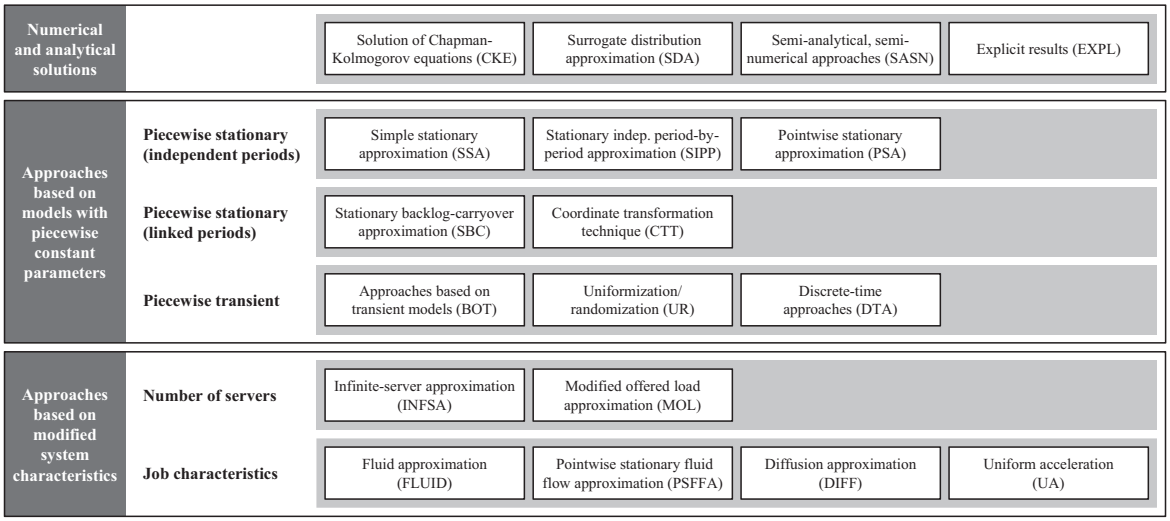

Fig. 1. Classification of approaches.

volumes and agents at inbound call centers, time-varying air traffic at airports, non-stationary truck arrival rates at container terminals, and cyclic message volumes in IT systems. Because these time-dependent parameter changes can have a substantial impact on a queueing system's performance, they must be considered in the design and control of such systems.

In this paper, we classify performance evaluation methods for single-stage queueing systems with time-dependent but deterministic parameter changes. While such systems are also called non-stationary, time-varying, time-inhomogeneous, or non-homogeneous queueing systems, we solely use the term time-dependent queueing systems.

The analysis of time-dependent queueing systems has a long tradition dating back to Kolmogorov (1931) [131]. Since then, the practical relevance of such systems has stimulated increasing interest in various research areas, including mathematics, computer science, and operations management. Such an analysis itself is difficult since common relations for steady-state queueing systems, such as Little's law, must be reformulated [18].

The contribution of the present work is a survey and classification of the literature on performance evaluation approaches for time-dependent queueing systems and their applications. Additionally, links between different approaches are identified and discussed.

The remainder of this paper is organized as follows. Both, the scope and the classification scheme, are introduced in Section 2. In Section 3, approaches for the analytical treatment of time-dependent queueing systems are reviewed and classified according to the developed scheme. A visualization of links between the approaches and a review of numerical studies that compare several methods are provided in Section 4. Areas of application and their unique characteristics are described in Section 5. In Section 6, concluding remarks and areas for future research are provided.

2. Scope and classification scheme

The survey presented in this paper reviews and classifies approaches for the time-dependent performance evaluation of single-stage queueing systems without spatial dimension, known as point queues, that include

- abandonments and retrials,
- arrivals from an infinite population that are served individually by a single server or one of multiple parallel servers (for a treatment of finite source systems, see e.g. Alfa [7], Chung and Min [39], and references within),
- waiting rooms larger or equal to one (i.e., waiting or loss-waiting systems; for a recent but incomplete survey of time-dependent loss queues, see Alnowibet and Perros [12]),
- and deterministic system parameters that change over time (the transient analysis of systems with constant parameters is addressed, e.g., by Van de Coevering [220], Tarabia [211], and references within).

We survey approaches that allow for the performance analysis of arbitrary time instances. Discrete event simulation is also applied for time-dependent performance evaluation. However, it is associated with a simulation error. This error can be reduced by an increase in the number of replications at the price of increasing run times [167]. Moreover, structural system properties remain intractable. Thus, the survey comprises only approaches which do not require the generation of random numbers.

We identify three main categories of evaluation approaches: the first category comprises numerical and analytical solution approaches for systems of equations that describe the time-dependent behavior of a queueing system (Section 3.1); the second category includes approaches that assume piecewise constant parameters and that apply stationary or transient models (Section 3.2); and the third category includes approximation methods that modify the number of servers or properties of the processed jobs (Section 3.3). Fig. 1 presents our classification scheme including these categories and all evaluation approaches reviewed in this work. In the corresponding sections, each approach is described in terms of its key idea, its chronological development, and its advantages and limitations. These descriptions include only references that develop or methodologically extend an approach. All surveyed references together with the characteristics of the analyzed queueing systems are listed in Tables 2,3, and 5–10. For each reference that considers several queueing systems, the characteristics of the most general one are given. The references are sorted chronologically for each approach. The notation used in the following sections is provided in Table 1.

The development of approaches for the performance evaluation is often driven by real-world problems. Hence, many articles include both an evaluation approach and its application to a real-world problem. The classification according to the area of application considers the references that include a detailed description of a specific application accompanied by a numerical study. The reviewed applications of the approaches are divided into the areas