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The set of fuzzy rational numbers and flexible querying

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Abstract

A fuzzy bag is a bag in which each occurrence of an element is associated with a grade of membership. This notion can be viewed as a generalization of the concepts of set, fuzzy set and bag. The set of fuzzy integers ($\mathbb{N}_f$) provides a general characterization in which all these different concepts are treated in a uniform way and can then be composed. In the field of databases, the use of fuzzy bags is motivated by their ability to manage both quantities and preferences. However, $\mathbb{N}_f$ becomes too restricted a framework when dealing with queries based on difference or division operations. So, a more general structure based on the set of fuzzy relative integers ($\mathbb{Z}_f$) in which exact differences can be performed, has been first developed. In this paper, we carry on with this approach and we extend $\mathbb{Z}_f$ to the set of fuzzy rational numbers ($\mathbb{Q}_f$). This context leads to define a closed system of multiplicative operations and allows to perform exact divisions. Applied to flexible querying of databases, $\mathbb{Q}_f$ and the notion of division on fuzzy numbers allow to generalize the relational division. They define a sound basis for queries involving ratios between quantities.

Keywords: Fuzzy integer; Fuzzy cardinality; Fuzzy rational number; Fuzzy bags; Bags; Fuzzy sets; Flexible querying

1. Introduction

An issue in extending database management functionalities is to increase the expressiveness of query languages. Flexible querying [3] enables users to express preferences inside requirements. Fuzzy set theory offers a general framework for dealing with flexible queries and priorities inside compound queries. The answers to such queries are then qualified and rank-ordered. Besides, the bag type [1,2], which offers

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the capability of managing quantities (numbers of occurrences of data items), plays an important role in databases [15,18] and data models (relational or object oriented) have been designed to support it. Systems taking into account both flexible queries and bags motivate the use of fuzzy bags. For example, a fuzzy bag can be obtained when some attributes are removed from a fuzzy set of tuples. This is illustrated by the query: *find the salaries of young employees* which requires a projection (salary) of a fuzzy set of persons (the young employees) and delivers a fuzzy bag. As several employees may have the same salary, the collection of salaries returned may contain duplicates. Moreover, a given salary occurrence is associated with a more or less young employee and thus satisfies more or less the criterion "to be the salary of a young employee". Consequently, the different salaries returned by the query have to be managed both quantitatively and qualitatively thanks to a fuzzy bag which represents the distribution of the salaries of young employees.

Our research aim is to devise new structures capable of dealing with quantification and preferences on data. These models can then be used for extending elementary query operators that provide a sound basis for designing high level query languages such as OQL or SQL. So, we are mainly concerned with the study of flexible querying of databases and we follow a pragmatic, application domain-driven approach. But, it is worth mentioning that our investigations have a larger scope than the field of databases and many other potential application domains could also benefit from fuzzy bags, such as fuzzy data mining, summarization of data or fuzzy information retrieval.

Fuzzy bags and some of their operators have been defined by Yager in [30,31] and complementary studies have been carried out in [7,8,9,17,20,21]. In [23,27], we have proposed a new approach for building fuzzy bags so as to introduce operators compatible with both bags and fuzzy sets. Hence, we have shown that fuzzy bags can be viewed as a generalization of fuzzy sets thanks to the consideration of an order structure over the unit interval. Their characteristic function is then defined from a universe U to the set of conjunctive fuzzy natural integers ($\mathbb{N}_f$). However, in this context, the difference operation between two bags A and B cannot always be computed. This problem comes from the fact that the fuzzy bag model considered so far is based on positive fuzzy integers. It is the reason why the set of fuzzy relative integers ($\mathbb{Z}_f$) was constructed. In such a framework, as discussed in [25], the difference $A - B$ of two fuzzy bags is always defined.

This paper, situated in the continuation of these works, aims at extending $\mathbb{Z}_f$ to $\mathbb{Q}_f$, the set of fuzzy rational numbers. This context leads to define a closed system of multiplicative operations and to perform exact divisions. The role of these arithmetic structures is illustrated in the field of flexible querying of databases where $\mathbb{Q}_f$ and the notion of division on fuzzy numbers allow to generalize the relational division or to define a sound basis for queries calling on ratios between quantities.

The rest of this paper is organized as follows. Sections 2 and 3 recall some key notions which constitute the background of the new contributions developed in Sections 4 and 5. Thus, Section 2 introduces the concepts of fuzzy bags and fuzzy natural integers. The main definitions and operators are recalled in Subsections 2.1 and 2.2. In Section 3, the extension of $\mathbb{N}_f$ to $\mathbb{Z}_f$ and the concept of a fuzzy bag defined on $\mathbb{Z}_f$ are briefly discussed. Next, Section 4 is devoted to a complementary study extending $\mathbb{Z}_f$ to $\mathbb{Q}_f$. Main definitions, operators and algebraic properties are first analyzed, then the exact division on $\mathbb{Q}_f$ and its different approximations on $\mathbb{N}_f$ or $\mathbb{R}$ are more specifically considered. Last, the usefulness of these propositions is emphasized in the database domain. Thus, in Section 5, we first study some generalizations of relational applications thanks to approximate divisions, then we illustrate the interest of $\mathbb{Q}_f$ when dealing with a query such as *what is the average salary of a fuzzy set of young employees*? Or when evaluating grades of inclusion and similarity measures based on divisions of fuzzy cardinalities.