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# An answer to an open problem on triangular norms

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**Abstract**

In this paper, we shall give an answer to show that an open problem, which is collected by Klement et al. in "Problems on triangular norms and related operators" [Fuzzy Sets and Systems 145 (2004) 471–479], has a negative solution.


*Keywords:* Triangular norms; Conditionally cancellative t-norms; Archimedean t-norms

## 1. Introduction

The following problem was posed by Pap [7].

**Problem 1.** *Let $T$ be a cancellative* t*-norm which is continuous at the point* (1,1). *Is $T$ necessarily continuous*?

A negative answer was given by Budinčević and Kurilić [1]. Moreover, there are non-continuous cancellative t-norms which are left-continuous [8]. On the other hand, for an Archimedean t-norm, its left-continuity is equivalent to its continuity [6].

In [4] several open problems on triangular norms and related operators were collected, which were raised during the 24th Linz seminar on fuzzy theory "Triangular norms and related operators in many-valued logics" held in February 2003. Two of them are stated as follows:

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**Problem 2.** *Let T be a conditionally cancellative* t-*norm which is continuous at the point* (1,1) *and with zero divisors. Is T necessarily continuous*?

**Problem 3.** *Let T be a conditionally cancellative left-continuous* t-*norm which has zero divisors. Is T necessarily continuous*?

In Section 2, we answer Problem 2.

Note that, for t-norms without zero divisors, Problem 2 is exactly the solved Problem 1 [1]. There are examples of non-continuous cancellative t-norms which are left continuous. One example for this is the following [8]: we recall that each $(x, y) \in (0, 1]^2$ is in a one-to-one correspondence with a pair $((x_n)_{n\in N}, (y_n)_{n\in N})$ of strictly increasing sequences of natural numbers given by the unique infinite dyadic representations

$$x = \sum_{n=1}^{\infty} \frac{1}{2^{x_n}} \quad \text{and} \quad y = \sum_{n=1}^{\infty} \frac{1}{2^{y_n}}$$

of the numbers $x$ and $y$, respectively. Using this notion, then the function $T: [0, 1] \times [0, 1] \to [0, 1]$ given by

$$T(x, y) = \begin{cases} \sum\limits_{n=1}^{\infty} \dfrac{1}{2^{x_n+y_n-n}} & \text{if } (x, y) \in (0, 1]^2, \\ 0 & \text{otherwise,} \end{cases}$$

is a t-norm which is cancellative (thus without zero divisors), left continuous on $[0, 1]^2$, but discontinuous in each point $(x, y) \in (0, 1)^2$, where at least one coordinate is a dyadic rational number.

For basic notions and properties we refer the reader to Refs. [3,5].

**Definition 1.** A triangular norm (briefly t-norm) is a binary operation $T$ on the unit interval [0, 1] which is commutative, associative, non-decreasing and has 1 as neutral element, i.e., it is a function $T: [0, 1] \times [0, 1] \to [0, 1]$ such that for all $x, y, z \in [0, 1]$:

(T1) $T(x, y) = T(y, x)$,

(T2) $T(x, T(y, z)) = T(T(x, y), z)$,

(T3) $T(x, y) \leqslant T(x, z)$ whenever $y \leqslant z$,

(T4) $T(x, 1) = x$.

**Remark 1.** Let $T$ be a t-norm. Then the conditions (T3) and (T4) in Definition 1 imply that $T(x, y) \leqslant x \wedge y$, where $x \wedge y := \min(x, y)$.

**Definition 2.** A t-norm $T$ is called conditionally cancellative if the equality $T(x, y) = T(x, z) > 0$ implies $y = z$.