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Slacks-based inefficiency approach for general networks with bad outputs: An application to the banking sector

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ABSTRACT

In this paper the efficiency assessment of general networks of processes that produce both desirable and undesirable outputs is addressed. This problem arises in many contexts (e.g. transportation, energy generation, etc.). A general networks slacks-based inefficiency (GNSBI) measure can be computed using a simple linear program that takes into account the weak disposability of the bad outputs. The slacks-based inefficiency (SBI) of each process is also calculated. Target values for all inputs, outputs (both desirable and undesirable) and even intermediate products are also provided. The proposed approach is rather general and can accommodate many different network topologies and returns to scale assumptions. Two applications to the banking sector are presented: one to assess banks efficiencies and another to assess bank branches.

1. Introduction

Data Envelopment Analysis (DEA) is a well-known non-parametric methodology for assessing the relative efficiency of comparable Decision Making Units (DMUs). It is a technique that has been used in many different sectors (e.g. [37,32,19]). A DMU is sometimes modelled as a single process that consumes inputs and produces outputs. However, there are so-called network DEA (NDEA) models that instead of considering just a single process, consider the system as a network of interrelated processes, each one with its own inputs and outputs and also with intermediate products, generated and consumed within the system [14]. The number and sophistication of NDEA models has increased significantly and some of them have had a significant impact (e.g. [35,27,8,26,29,48,49,16,12,9,34,5]). The number of applications has also grown substantially, especially in transportation (e.g. [55,39]), supply chains [10,47], banking (e.g. [4,6,42]), tourism (e.g. [21,24]) and sports (e.g. [33,44]).

Some of the existing NDEA applications have had to deal with the existence of bad outputs. To the best of the author's knowledge, Kordrostami and Amirteimoori [30] were the first to consider undesirable factors in NDEA. They considered a multistage system and proposed a multiplier formulation that subtracted undesirable outputs from desirable outputs to compute the virtual output (and added desirable inputs to normal inputs to compute the virtual input). Later, Hua and Bian [23] extended this multiplier formulation approach to any network topology, not necessarily in series. Bi and Tao [7] used a leader-follower, two-stage approach to assess the ecological efficiency of paper mills using biochemical oxygen demand (BOD) as a bad output.

Fukuyama and Weber [16] and Akther et al. [2] used a network slacks-based inefficiency measure (NSBI) to assess banks modelled as a two-stage with bad outputs. Weak disposability (WD) of the undesirable output (namely non-performing loans) was assumed. Hosseinzadeh Lofti and Hasanpour [22] proposed a weighted directional distance NDEA model that considered undesirable factors as well as non-discretionary data but not weak disposability. Ashrafi and Jafaar [3] also considered a two-stage system and used a weighted additive efficiency decomposition (WAED) approach to assess bank branches considering a bad output.

Lozano et al. [39] used a directional distance function on a two-stage system to assess airports efficiency including flights delays as bad outputs. They considered that the undesirable outputs were WD as per Färe and Grosskopf [15], which leads to non-linear programs that not always can be linearised. Song et al. [46] used a two-stage network slacks-based measure of efficiency (NSBM) to assess the environmental efficiency of Chinese provinces in terms of the wastewater they generate and discharge.

Maghbouli et al. [41] proposed two different two-stage models, a leader-follower approach to handle undesirable intermediate products and a centralised NDEA approach for the case of undesirable final outputs. Interestingly, they considered WD of the undesirable outputs, using the WD approach proposed in Kuosmanen [31], which always results in a linear program. Ebrahimnejad et al. [13] used a three-stage network with two bad outputs in the final stage. They did not consider, however, WD of the bad outputs. Finally, Mirhedayatian

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et al. [43] proposed a NSBM approach to study multistage systems considering dual-role factors, undesirable outputs and fuzzy data. They applied it to assess the efficiency of green supply chains consisting of four echelons (supplier, producer, distributor and customer).

Note that, in the above review on network DEA approaches involving undesirable outputs (NDEA-UO), we have only included those works that treat bad outputs specifically. There are also other researchers that have treated them as inputs (e.g. [42]) or as normal outputs after carrying out an appropriate transformation (e.g. [52]). Note also that most of the NDEA-UO approaches in the literature deal with two-stage systems and many of them do not take into account WD.

In this paper the NSBI approach in Fukuyama and Weber [16] is extended to general networks of processes, using, for the undesirable outputs, the WD approach proposed in Kuosmanen [31]. In addition to the overall GNSBI measure, a specific SBI score is computed for each individual process. The approach is rather general, in terms of the size and topology of the networks considered as well as the corresponding returns to scale (RTS) assumptions of the different processes. As illustration of the versatility and usefulness of the proposed approach two different applications to the banking sector are discussed.

A major contribution of the paper is, thus, the possibility of easily implementing the SBI measure for any network of processes. Note that, although most NDEA applications so far have involved relatively simple two-stage systems, researchers have started to consider more complex topologies of processes (e.g. [49,44]). This is a trend that is likely to continue in the future and demands the development of adequate general and flexible NDEA models. It is towards that end that the present paper is aimed.

The structure of the paper is the following. The proposed GNSBI approach is presented in Section 2. Section 3 reviews the literature on NDEA applications to the banking sector. In Section 4, two different banking sector applications are considered: one of them dealing with banks efficiency and the other with bank branches efficiency. Finally, Section 5 summarises and concludes.

2. General networks slacks-based inefficiency approach (GNSBI)

In order to formulate a DEA model for general networks of processes the notation used can be of a great help if it is chosen appropriately. The one that will be presented below was first used in Lozano [38] and has proven very convenient and helpful (e.g. [39]). In Appendix A, a complete list of the notations used is presented. Thus, let us consider that there are n DMUs, all of which consist of the same processes, with the same interrelationships among them. Let P be the number of processes. Let $I(p)$ be the set of inputs of process p and $P_I(i)$ the set of processes that consume the input i. For each $i \in I(p)$, let x_{ij}^{p} denote the observed amount of input i consumed by process p of DMU j.

Similarly, let $O(p)$ (respectively $B(p)$) be the set of desirable outputs (respectively bad outputs) of process p. For each $k \in O(p)$ and each $q \in B(p)$, let y_{kj}^{p} and b_{qj}^{p} denote the observed amount of desirable output k and bad output q, respectively, produced by process p of DMU j. Let $P_O(k)$ be the set of processes that produce the final output k and $P_B(b)$ the set of processes that produce the undesirable output b. Let also $P_B = \bigcup_b P_B(b) = \{p : B(p) \neq \varnothing\}$ the set processes that generate any type of bad outputs. This set is needed because those are the processes for which WD is considered, which will have an effect in the formulation.

In addition to these inputs, desirable outputs and bad outputs, let us assume that there exist a number of intermediate products which are generated by some processes and consumed by others. Thus, let $P^{out}(r)$ be the set of processes that generate the intermediate product r and $P^{in}(r)$ the set of processes that consume the intermediate product r. For each $p \in P^{out}(r)$ let z_{rj}^{p} be the observed amount of intermediate product r generated by process p of DMU j and, analogously, for each $p \in P^{in}(r)$ let z_{rj}^{p} the observed amount of intermediate product r consumed by process p of DMU j. Finally, let $R^{out}(p)$ and $R^{in}(p)$ be the sets of the intermediate products produced and consumed, respectively, by a certain process p. Note that, because the intermediate products are endogenously produced and consumed, we assume that

$$\sum_{p \in P^{out}(r)} z_{rj}^{p} = \sum_{p \in P^{in}(r)} z_{rj}^{p} \quad \forall r \ \forall j \tag{1}$$

i.e. the total observed amount of an intermediate product r consumed by the different processes of a DMU j is equal to the total observed amount of an intermediate product r produced by the processes of DMU j.

About the RTS of the different processes, let P_{VRS} the set of processes that exhibit Variable Returns to Scale (VRS), P_{NIRS} the set of processes that exhibit Non-Increasing Returns to Scale (NIRS) and P_{NDRS} those that exhibit Non-Decreasing Returns to Scale (NDRS). The rest of processes are assumed to exhibit Constant Returns to Scale (CRS), i.e. $P_{CRS} = \{p : p \notin P_{NIRS} \ \cap \ p \notin P_{VRS}\}$.

The slacks-based inefficiency measure uses the components of a given direction vector $\mathbf{g} = (g_i^x, g_k^y, g_q^b)$ to normalise the different slacks [16]. Also, same as in the NSBM approach [48] or in the NRAM approach [5] we will assume the possibility of using different weights $\alpha^p \in [0,1]$ reflecting the importance of the different processes. These weights are assumed to be normalised, i.e. $\sum_p \alpha^p = 1$

As for the decision variables of the model, let

s_i^{p-}	slack of input i of process p
s_k^{p+}	slack of desirable output k of process p
$s_q^{p\#}$	slack of bad output q of process p
λ_j^p	intensity variable used for process p of DMU j when computing linear combinations of the observed DMUs
μ_j^p	auxiliary WD intensity variable used for process p of DMU j when computing linear combinations of the observed DMUs. This auxiliary WD intensity variable affects only the inputs of those processes that produce bad outputs and its aim is to take into account the WD of those bad outputs (see [31,41]). This is a very convenient way of modelling WD since it does not introduce non-linearities as it happens in the case of the WD approach of Färe and Grosskopf [15].

We are now ready to formulate the proposed general NSBI DEA model. The subindex 0 in model (2)–(9), and elsewhere in the paper, refers to the DMU whose efficiency is being assessed. Each DMU is projected separately, which means that the model is solved for each DMU in turn. Hence, in order to indicate the DMU being projected the subindex 0 is used. The GNSBI measure of efficiency of DMU 0 is

$$\text{GNSBI}_0 = \text{Max} \sum_p \frac{\alpha^p}{|I(p)| + |O(p)| + |B(p)|} \cdot \left(\sum_{i \in I(p)} \frac{s_i^{p-}}{g_i^x} + \sum_{k \in O(p)} \frac{s_k^{p+}}{g_k^y} + \sum_{q \in B(p)} \frac{s_q^{p\#}}{g_q^b} \right) \tag{2}$$

subject to

$$\sum_j \left(\lambda_j^p + \mu_j^p \right) \cdot x_{ij}^{p} = x_{i0}^{p} - s_i^{p-} \qquad \forall p \ \ \forall i \in I(p) \tag{3}$$

$$\sum_j \lambda_j^p \cdot y_{kj}^{p} = y_{k0}^{p} + s_k^{p+} \qquad \forall p \ \ \forall k \in O(p) \tag{4}$$