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Combining analytical hierarchy process and Choquet integral within non-additive robust ordinal regression ☆

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ABSTRACT

We consider multiple criteria decision aiding in the case of interaction between criteria. In this case the usual weighted sum cannot be used to aggregate evaluations on different criteria and other value functions with a more complex formulation have to be considered. The Choquet integral is the most used technique and also the most widespread in the literature. However, the application of the Choquet integral presents two main problems being the necessity to determine the capacity, which is the function that assigns a weight not only to all single criteria but also to all subset of criteria, and the necessity to express on the same scale evaluations on different criteria. While with respect to the first problem we adopt the recently introduced Non-Additive Robust Ordinal Regression (NAROR) taking into account all the capacities compatible with the preference information provided by the DM, with respect to the second one we build the common scale for the considered criteria using the Analytic Hierarchy Process (AHP). We propose to use AHP on a set of reference points in the scale of each criterion and to use an interpolation to obtain the other values. This permits to reduce considerably the number of pairwise comparisons usually required by the DM when applying AHP. An illustrative example details the application of the proposed methodology.

1. Introduction

In Multiple Criteria Decision Aiding (MCDA) problems (see [42] for an accessible guide to MCDA and [20] for a comprehensive collection of state of the art surveys), a set of alternatives $A=\{a,b,c,\ldots\}$ is evaluated on a set of evaluation criteria $G=\{g_1,\ldots,g_n\}$ (sometimes, for the sake of simplicity and slightly abusing of the notation, we refer to the criteria with their indices, i.e. we shall write $i\in G$, instead of $g_i\in G$). Typical MCDA problems are choice, sorting and ranking. Choice problems consist of choosing a subset (possibly composed of one element only) $A^*\subseteq A$ of alternatives considered the best; sorting problems consist of assigning each alternative to one or more predefined and preferentially ordered contiguous classes, while ranking problems consist of partially or completely ordering all alternatives from the best to the worst.

Looking at the evaluations of the alternatives on the criteria, without taking into account further preference information and any preference model, it could be only observed if the dominance relation is fulfilled by some pairs of alternatives.[1] In general, the dominance relation provides really poor information and leaves many alternatives incomparable. For this reason, to get more precise recommendations on the problem at hand, there is the necessity to aggregate the evaluations of the alternatives on the considered criteria through some appropriate preference model representing the preferences of the Decision Maker (DM). In the literature the most well-known aggregation methods are the Multi-Attribute Value Theory (MAVT) [45] and the outranking methods (for ELECTRE methods see [21,23,54] and for PROMETHEE methods see [11,12]). MAVT assigns to each alternative a a real number $U(a)$ being representative of the degree of desirability of a with respect to the problem at hand, while outranking methods are based on an outranking relation being a binary relation S on the set of alternatives A, such that aSb means that a is at least as good as b.

Both family of methods are based on the mutual preference independence between criteria [45,64] but, in many real world decision making problems, the evaluation criteria are not

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[1] An alternative a dominates an alternative b if the evaluations of a are at least as good as the evaluations of b on all criteria and better for at least one criterion.

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independent but interacting. For instance, suppose the DM likes sport cars and she wants to buy a car taking into account the criteria price, maximum speed and acceleration. In this case, maximum speed and acceleration can be considered negatively interacting criteria while maximum speed and price can be considered positively interacting criteria. In fact, on one hand, even if maximum speed and acceleration are very important for a DM liking sport cars, in general cars with a high maximum speed have also a good acceleration and, therefore, the comprehensive importance of the two criteria considered together should be smaller than the sum of the importance of the two criteria considered alone. On the other hand, a car with a high maximum speed has often also a high price and, therefore, a car with a high maximum speed and a moderate price is very well appreciated. Consequently, the comprehensive importance of these two criteria considered together should be greater than the sum of the importance of the two criteria considered alone.

In such cases, the mutual preference independence can be violated because, for example, due to the positive interaction between maximum speed and price, at a given level of price, one can prefer one combination of maximum speed and acceleration, while at another level of price one can prefer another combination of maximum speed and acceleration. Observe however that the violation of preference independence does not imply that the considered family of criteria is no more consistent. Indeed, consistency [54] refers to the requirements of *monotonicity*, that is, when improving the evaluations on considered criteria the overall evaluation of an alternative cannot be deteriorated, *exhaustivity*, that is, all the relevant criteria are considered, and *non-redundancy*, that is, no criterion can be removed without losing the representation of a relevant point of view. Monotonicity, exhaustivity and non-redundancy can continue to be satisfied also when preference independence does not hold. For instance, in the didactic example of Section 2, we show how reasonable can be the overall evaluations of students obtained aggregating scores in different subjects by the Choquet integral [14] rather than by the weighted sum. If the problem is correctly formulated, aggregation through Choquet integral satisfies monotonicity, exhaustivity and non-redundancy even if, as explained in the example, it does not satisfy preference independence.

Interaction between criteria and violation of the preference independence are well known in MCDA (see e.g. [7,19,26,44]). In the following, we briefly survey several methods handling with the interaction between criteria. Considering the utility functions as aggregation methods, the multilinear utility function [45] and the UTA^{GMS}-INT [38] are reported in the literature. The first one aggregates performances on considered criteria through a weighted sum of products of marginal utilities corresponding to single criteria, over all subsets of criteria, while UTA^{GMS}-INT is based on enriched additive value functions that add some further terms representing interaction between criteria to the usual sum of marginal utility functions. In Artificial Intelligence (AI), interaction between criteria has been recently considered through GAI-networks [30] as well as through UCP-networks [10], that are based on the idea of Generalized Additive Independence (GAI) decomposition [25]. Positive and negative interaction between criteria has been taken into account also in outranking methods such as ELECTRE [24] and PROMETHEE [15]. Another method that takes into consideration the interaction between criteria is the Analytical Network Process (ANP) [58]. In this case, interaction between criteria is one of the possible results of interdependencies and network between goals, criteria and alternatives. Observe that very specific interactions between criteria can be considered within ANP. For instance, ANP can model interactions that depend on the considered alternatives. This is the case of a positive interaction between criteria "price" and "maximum speed" for evaluating an economic car, which is not the case for a sport car. Considering interaction between criteria that can change from an alternative to another is not possible with the Choquet integral for which interaction between criteria holds in the same way for all the alternatives. However, the price to pay for such so fine modeling is an increased amount of preference information that can be difficult to supply for the DM.

Even if all cited methods are able to deal with the interaction between criteria, the most well-known methodologies in the literature are the non-additive integrals, such as the Choquet integral (see [14] for the original Choquet integral and [31] for the application of the Choquet integral in MCDA), the Sugeno integral [63] and the generalizations of the Choquet integral, that are the bipolar Choquet integral [34] or the level dependent Choquet integral [36]. The basic idea of these approaches is that the interaction between criteria can be modeled through a capacity, called also fuzzy measure, assigning a weight not only to each criterion but also to each subset of criteria.

In this paper, we shall consider the Choquet integral because, currently, it is the most adopted methodology to deal with interactions between criteria for its manageability (for example, we shall see that we can use linear programming to determine capacities compatible with DM's preferences) and for the meaningfulness of its preference parameters, namely the capacity that becomes understandable and intelligible even for the nonexpert DM using some specific techniques such as the Möbius representation, the Shapley index and the interaction index.

Even if it is theoretically appealing, the application of the Choquet integral, as well as the application of all non-additive methods mentioned above, involves some problems related to

(1) the determination of the capacity representing the interaction between criteria,
(2) the construction of a common scale permitting comparisons between evaluations on different criteria.

To handle point (1), we propose to use the Non-Additive Robust Ordinal Regression (NAROR) [4] that considers the whole set of capacities compatible with the preference information provided by the DM while, to handle point (2) we propose to use the Analytic Hierarchy Process (AHP, [56,57]). Let us spend some words to give the intuition behind our proposal. We shall give more details on how to deal with the two mentioned problems and on the reasons of combining them together in the following sections of the paper.

In any MCDA problem, a decision model has to be built to produce a recommendation and its preference parameters (weights, thresholds, value functions and so on) have to be determined. This is usually done in cooperation with the DM, who can give directly the preference parameters or, instead, can supply some preference information, for example in terms of preference pairwise comparison of some alternatives, from which preference parameters can be induced. In the case of the Choquet integral model, the preference parameters to be fixed are the weights that the capacity assigns to each one of the 2^n subsets of a family of n criteria (for example $2^{10} = 1024$ weights for a family of 10 criteria). Due to this so huge number of parameters, very often the values assigned by the capacity to the subset of criteria are not asked directly to the DM and, consequently, several methodologies have been proposed to determine a capacity compatible with the preference information provided by the DM. For example, in [34] four different approaches are presented to deal with this problem; however there is no general suggestion about which one to adopt. In these conditions, it seems very wise to take into account not one among the many capacities compatible with the DM's preference information, but, instead, the whole set of capacities compatible with the available preference information. This is the aim of the