Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Fuzzy Sets and Systems 151 (2005) 581–599
FUZZY sets and systems
www.elsevier.com/locate/fss

Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations

Barnabás Bede*, Sorin G. Gal

Department of Mathematics, University of Oradea, Str. Armatei Romane no. 5, 410087 Oradea, Romania

Received 14 November 2003; received in revised form 25 March 2004; accepted 2 August 2004
Available online 11 September 2004

Abstract

The usual concept of differentiability of fuzzy-number-valued functions, has the following shortcoming: if c is a fuzzy number and $g : [a, b] \to \mathbb{R}$ is an usual real-valued function differentiable on $x_0 \in (a, b)$ with $g'(x_0) \leqslant 0$, then $f(x) = c \odot g(x)$ is not differentiable on x_0. In this paper we introduce and study generalized concepts of differentiability (of any order $n \in \mathbb{N}$), which solves this shortcoming. Newton–Leibnitz-type formula is obtained and existence of the solutions of fuzzy differential equations involving generalized differentiability is studied. Also, some concrete applications to partial and ordinary fuzzy differential equations with fuzzy input data of the form $c \odot g(x)$, are given.
© 2004 Elsevier B.V. All rights reserved.

Keywords: Fuzzy-number-valued functions; Generalized differentiability; Fuzzy differential equations; Fuzzy partial differential equations

1. Introduction

The H-derivative of a fuzzy-number-valued function was introduced in [13] and it is studied in several papers (see e.g. [1,17]). In [14] is defined the Hukuhara derivative of a fuzzy-number-valued function and fuzzy initial value problem is studied. This derivative has its starting point in the Hukuhara derivative of multivalued functions. Differential equations in fuzzy setting are a natural way to model uncertainty of dynamical systems. There are different approaches to this very quickly developing area of fuzzy analysis. Let us mention some of them. First approach uses the above-mentioned H-derivative or its generalization,

* Corresponding author.
E-mail addresses: bbede@uoradea.ro (B. Bede), galso@uoradea.ro (S.G. Gal).

0165-0114/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.fss.2004.08.001

the Hukuhara derivative. Under this setting are mainly obtained existence and uniqueness theorems for the solution of a fuzzy differential equation (see e.g. [11,12,14,18,15], etc.). This approach has the disadvantage that it leads to solutions with increasing support, fact which is solved by interpreting a fuzzy differential equation as a system of differential inclusions (see e.g. [10,6]). But this last mentioned approach has at its turn some shortcomings. The main shortcoming is that one cannot talk about the derivative of a fuzzy-number-valued function, since a fuzzy differential equation is directly interpreted with the help of differential inclusions without having a derivative. Also solutions are not necessarily fuzzy-number-valued functions. Another approach can be found in [16], which provides solutions only in the class of pyramidal fuzzy numbers. In [5] are presented other two methods for solving fuzzy differential equations. These last two approaches suffer the same disadvantage as the method of differential inclusions, i.e. the derivative has no meaning.

In this paper we solve this shortcoming by introducing the weakly generalized differential of a fuzzy-number-valued function.

The study of usual known concept of differentiability of fuzzy-number-valued functions (see e.g. [1] or [17]) shows us that if $g:(a,b)\to\mathbb{R}$ is differentiable on $x_0\in(a,b)$ with $g'(x_0)<0$, then $f(x)=c\odot g(x)$, $\forall x\in(a,b)$, where c is a fuzzy number, is not in general differentiable on x_0. To solve this shortcoming, in Section 3 we introduce and study a new generalized concept of differentiability which extends the present concept and which allows us to have $f'(x)=c\odot g'(x)$, for all $x\in(a,b)$ when g is differentiable. Also, higher order concepts of differentiability are considered such that to have $f^{(n)}(x)=c\odot g^{(n)}(x)$, for all $x\in(a,b)$, $(n\geqslant 1)$, when exists $g^{(n)}(x)\in\mathbb{R}$.

This generalization allows us to solve in Sections 4 and 5, in a simple way, some higher order partial and ordinary fuzzy differential equations, whose fuzzy input data (coefficients) are of the form of above function f, i.e. products of fuzzy numbers with classical real-valued functions.

2. Preliminaries

Given a set $X\neq\emptyset$, a fuzzy subset of X is a mapping $u:X\to[0,1]$ (see [19]).

Let us denote by $\mathbb{R}_{\mathcal{F}}$ the class of fuzzy subsets of the real axis (i.e. $u:\mathbb{R}\to[0,1]$) satisfying the following properties:

(i) $\forall u\in\mathbb{R}_{\mathcal{F}}$, u is normal, i.e. $\exists x_0\in\mathbb{R}$ with $u(x_0)=1$;
(ii) $\forall u\in\mathbb{R}_{\mathcal{F}}$, u is convex fuzzy set (i.e. $u(tx+(1-t)y)\geqslant\min\{u(x),u(y)\}$, $\forall t\in[0,1]$, $x,y\in\mathbb{R}$);
(iii) $\forall u\in\mathbb{R}_{\mathcal{F}}$, u is upper semicontinuous on $\mathbb{R}$;
(iv) $\overline{\{x\in\mathbb{R};\ u(x)>0\}}$ is compact, where $\overline{A}$ denotes the closure of A.

Then $\mathbb{R}_{\mathcal{F}}$ is called the space of fuzzy numbers (see e.g. [8]). Obviously, $\mathbb{R}\subset\mathbb{R}_{\mathcal{F}}$. Here $\mathbb{R}\subset\mathbb{R}_{\mathcal{F}}$ is understood as $\mathbb{R}=\{\chi_{\{x\}};\ x$ is usual real number$\}$. For $0<r\leqslant 1$, denote $[u]^r=\{x\in\mathbb{R};u(x)\geqslant r\}$ and $[u]^0=\overline{\{x\in\mathbb{R};\ u(x)>0\}}$. Then it is well-known that for each $r\in[0,1]$, $[u]^r$ is a bounded closed interval. For $u,v\in\mathbb{R}_{\mathcal{F}}$, and $\lambda\in\mathbb{R}$, the sum $u\oplus v$ and the product $\lambda\odot u$ are defined by $[u\oplus v]^r=[u]^r+[v]^r$, $[\lambda\odot u]^r=\lambda[u]^r$, $\forall r\in[0,1]$, where $[u]^r+[v]^r$ means the usual addition of two intervals (subsets) of $\mathbb{R}$ and $\lambda[u]^r$ means the usual product between a scalar and a subset of $\mathbb{R}$ (see, e.g. [8,17]).

Defining $D:\mathbb{R}_{\mathcal{F}}\times\mathbb{R}_{\mathcal{F}}\to\mathbb{R}_+\cup\{0\}$ by $D(u,v)=\sup_{r\in[0,1]}\max\{|u_-^r-v_-^r|,|u_+^r-v_+^r|\}$, where $[u]^r=[u_-^r,u_+^r]$, $[v]^r=[v_-^r,v_+^r]$, the following properties are well-known (see e.g. [9] or [17]):

$D(u\oplus w,v\oplus w)=D(u,v)$, $\forall u,v,w\in\mathbb{R}_{\mathcal{F}}$,
$D(k\odot u,k\odot v)=|k|D(u,v)$, $\forall k\in\mathbb{R}$, $u,v\in\mathbb{R}_{\mathcal{F}}$,

Download English Version:

https://daneshyari.com/en/article/10324156

Download Persian Version:

https://daneshyari.com/article/10324156

Daneshyari.com