Omega 61 (2016) 35–48

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

A network DEA approach for series multi-stage processes ☆

Dimitris K. Despotis *, Dimitris Sotiros, Gregory Koronakos

Department of Informatics, University of Piraeus, 80, Karaoli and Dimitriou, 18534 Piraeus, Greece

ARTICLE INFO

Article history:
Received 16 February 2015
Accepted 6 July 2015
Available online 16 July 2015

Keywords:
Data envelopment analysis
Network DEA
Multi-stage processes
Composition paradigm

ABSTRACT

We present in this paper a general network DEA approach to deal with efficiency assessments in multi-stage processes. Our approach complies with the composition paradigm, where the efficiencies of the stages are estimated first and the overall efficiency of the system is obtained ex post. We use multi-objective programming as modeling framework. This provides us the means to assess unique and unbiased efficiency scores and, if required, to drive the efficiency assessments effectively in line with specific priorities given to the stages. A direct comparison with the multiplicative decomposition approach on data drawn from the literature brings into light the advantages of our method and some critical points that one should be concerned about when using the multiplicative efficiency decomposition.

1. Introduction

Data envelopment analysis (DEA) is a non-parametric technique for measuring the performance of decision-making units (DMUs) that use multiple inputs to produce multiple outputs. The underlying mathematical model is linear programming. The two basic DEA models, namely the CCR [4] and the BCC [1] models, have become standards in performance measurement under the assumptions of constant and variable returns-to-scale respectively. Conventional DEA deals with one-stage production processes, where the internal structure of the DMUs is not taken into account. The network DEA paradigm, on the other hand, refers to multi-stage processes, where the underlying structure, which indicates the flow of the intermediate measures among the stages, plays a key role in the efficiency assessment. Färe and Grosskopf [11] were among the first to study the efficiency in such processes, represented as network activity analysis models. Castelli et al. [2] provide a comprehensive categorized overview of models and methods developed for different multi-stage production configurations. Kao [15] provides a thorough classification of studies in network DEA, according to the type of the network structure and the model employed. The series and the parallel production processes are two characteristic process configurations studied extensively in the literature. As the latter is beyond the scope of this paper, the reader is referred to [9,12,13–15], where one can identify some links between parallel and series processes. In this paper we focus on multi-stage series production process. The first approaches to deal with the efficiency assessment in two-stage series processes is the *multiplicative decomposition approach* introduced by Kao and Hwang [17] and the *additive decomposition approach* introduced by Chen et al. [5]. Both approaches are based on the reasonable assumption, which ever since is consolidated in the literature, that the weights used for the intermediate measures are the same, no matter if these measures are considered as outputs of the first stage or inputs to the second stage. Liang et al. [20] and Cook et al. [6] studied the efficiency decomposition in two-stage processes using game theoretic concepts. Zhou et al. [22] approached the efficiency decomposition in simple two-stage processes as a Nash bargaining game. Li et al. [19] used a parametric approach to assess the efficiency of DMUs with extra inputs in the second stage, in the frame of the multiplicative approach. Kao et al. [18] used a multi-objective programming approach to the efficiency assessments in network structures. Extensions for multi-stage series processes are given in [2,12,15,16]. Recently, Despotis et al. [8] introduced the *composition paradigm* in two-stage network DEA. Unlike the efficiency decomposition approach, in the composition approach the efficiencies of the two stages are estimated first and the overall efficiency of the DMU is obtained ex post. A major advantage of the assessment method presented in [8] over the additive [5] and the multiplicative [17] methods is that the former provides unique and unbiased efficiency scores for two-stage processes. Its disadvantage, however, is that it cannot be readily extended in multi-stage series processes. This is an effect of the different orientations selected for the first and the second stage, which in fact was made to simplify the models and keep them within the field of linear programming (simplicity at the expense of generality).

In this paper we extend the composition paradigm in general series multi-stage processes, by proposing a multi-objective

☆This manuscript was processed by Associate Editor Lim.

* Corresponding author. Tel.: +30 210 4142315.
E-mail addresses: despotis@unipi.gr, ddespotis@gmail.com (D.K. Despotis).

http://dx.doi.org/10.1016/j.omega.2015.07.005
0305-0483/

programming approach. Without harming simplicity, our approach overcomes the lack of generality in [8], as long as our model and the solution method proposed can handle any type of series multi-stage process. Our developments makes the direct comparison of the new approach with the multiplicative method [17] possible and fruitful, in a manner that enables us to point out some critical issues that one should take into account when using the multiplicative decomposition method. Unlike the additive and the multiplicative decomposition methods, our new general approach secures the uniqueness of the efficiency scores. Moreover, the efficiency assessments are neutral, in the sense that no implicit priority is assumed for some stages over the others.

The paper is organized as follows. Section 2 is devoted to two-stage processes. We identify four distinct types of processes that cover all possible configurations. In Section 2.1 we unfold our modeling approach in detail with respect to the elementary two-stage process, which assumes that nothing but the external inputs to the first stage enters the system and nothing but the outputs of the second stage leaves the system. A thorough comparison of our method with the multiplicative approach [17] highlights the advantages of the former and points out some critical peculiarities of the latter. In Sections 2.2–2.4, we apply the same approach to other two-stage configurations. When case data are available in the literature, we compare the results obtained by our method with those from other methods. Otherwise, we provide the reader with synthetic data and the corresponding results for testing and validation. In Section 3 we extend our formulations in general multi-stage processes. Conclusions are drawn in Section 4.

2. Two-stage processes

In this section we develop our network DEA approach for the case of two-stage series processes. We follow the composition paradigm introduced in [8]. In the composition paradigm, as opposed to the decomposition approach (cf. [17,5]), the stage efficiencies are estimated without any a priori definition of the overall efficiency of the system. Once the stage efficiencies are estimated, the overall efficiency is computed a posteriori by aggregating the stage efficiencies additively or multiplicatively. We consider four types of processes that cover all possible two-stage series configurations, as depicted in Fig. 1.

Let us introduce the following basic notation:

$j \in J = \{1, \ldots, n\}$: The index set of the n DMUs.
$j_0 \in J$: Denotes the evaluated DMU.
$X_j = (x_{ij}, i = 1, \ldots, m)$: The vector of stage-1 external inputs used by DMU_j (all types).
$Z_j = (z_{pj}, p = 1, \ldots, q)$: The vector of intermediate measures for DMU_j (all types).

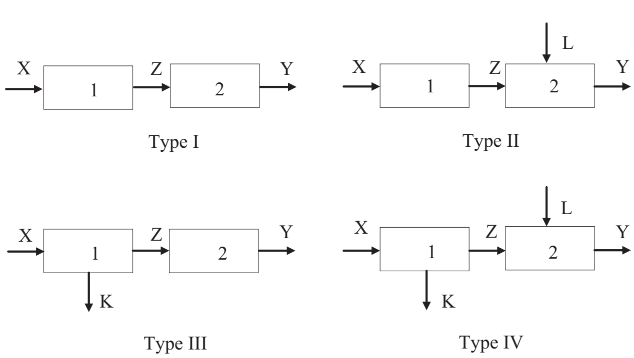

Fig. 1. The four types of series two-stage processes.

$Y_j = \left(y_{rj}, r = 1, \ldots, s\right)$: The vector of stage-2 final outputs produced by DMU_j (all types).
$L_j = (l_{dj}, d = 1, \ldots, a)$: The vector of stage-2 external inputs (types II and IV).
$K_j = (k_{cj}, c = 1, \ldots, b)$: The vector of stage-1 final outputs (types III and IV).
$\eta = (\eta_1, \ldots, \eta_m)$: The vector of weights for the stage-1 external inputs in the fractional model.
$v = (v_1, \ldots, v_m)$: The vector of weights for the stage-1 external inputs in the linear model.
$\varphi = (\varphi_1, \ldots, \varphi_q)$: The vector of weights for the intermediate measures in the fractional model.
$w = (w_1, \ldots, w_q)$: The vector of weights for the intermediate measures in the linear model.
$\omega = (\omega_1, \ldots, \omega_s)$: The vector of weights for the stage-2 outputs in the fractional model.
$u = (u_1, \ldots, u_s)$: The vector of weights for the stage-2 outputs in the linear model.
$\gamma = (\gamma_1, \ldots, \gamma_a)$: The vector of weights for the stage-2 external inputs.
$\mu = (\mu_1, \ldots, \mu_b)$: The vector of weights for the stage-1 final outputs.
e_j^o: The overall efficiency of DMU_j.
e_j^1: The efficiency of the first stage for DMU_j.
e_j^2: The efficiency of the second stage for DMU_j.
E_j^1: The independent efficiency score of the first stage for DMU_j.
E_j^2: The independent efficiency score of the first stage for DMU_j.

2.1. Type I structure

Consider the elementary case (Type I) where each DMU transforms some external inputs X to final outputs Y via the intermediate measures Z with a two-stage process, as depicted in Fig. 1. In this basic setting, nothing but the external inputs to the first stage enters the system and nothing but the outputs of the second stage leaves the system. Typically, the efficiency of the first and the second stage of a DMU j are defined as follows:

$$e_j^1 = \frac{\varphi Z_j}{\eta X_j}, \quad e_j^2 = \frac{\omega Y_j}{\varphi Z_j}$$

The overall efficiency of DMUj is defined as the ratio of the total virtual exogenous output to the total virtual exogenous input:

$$e_j^o = \frac{\omega Y_j}{\eta X_j}$$

Consider the basic input oriented CRS-DEA models that estimate the stage-1 and the stage-2 efficiency for the evaluated unit j_0 independently:

$$\begin{aligned} E_{j_0}^1 = \; & \max \frac{\varphi Z_{j_0}}{\eta X_{j_0}} \\ & \text{s.t.} \\ & \varphi Z_j - \eta X_j \le 0, \; j = 1, \ldots, n \\ & \eta \ge \varepsilon, \; \varphi \ge \varepsilon \end{aligned} \tag{1}$$

$$\begin{aligned} E_{j_0}^2 = \; & \max \frac{\omega Y_{j_0}}{\varphi Z_{j_0}} \\ & \text{s.t.} \\ & \omega Y_j - \varphi Z_j \le 0, \; j = 1, \ldots, n \\ & \varphi \ge \varepsilon, \; \omega \ge \varepsilon \end{aligned} \tag{2}$$

In order to link the efficiency assessments of the two stages, it is universally accepted that the weights associated with the intermediate measures are the same, no matter if these measures are