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Pareto–Koopmans efficiency and network DEA ☆

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ABSTRACT

Standard or black-box data envelopment analysis (DEA) evaluates the efficiency of the transformation of a DMU's exogenous inputs into its final outputs by ignoring what is going on in its divisions (sub-DMUs). To cope with this problem, network DEA (NDEA), which can provide adequate detail to management, has been developed and applied empirically. However, we show that some of the commonly used NDEA methods are inconsistent with the notion of Pareto–Koopmans efficiency. Since the original development of DEA, Pareto–Koopmans efficiency is a fundamental property used in DEA. From a Pareto–Koopmans efficiency perspective, therefore, we propose a two-phase NDEA approach that can provide information on both each DMU's overall (system) efficiency status and its divisions' efficiency scores. The proposed novel approach is developed based on the enhanced Russell graph model or equivalently the slacks-based model. We also propose several theorems and illustrate the proposed approach using two artificial numerical examples and a real-world data set.

1. Introduction

Standard or black-box data envelopment analysis (DEA) is a set of mathematical programming techniques for measuring the efficiency performance of decision making units (DMUs) that convert exogenous inputs into final outputs. In standard DEA, the internal production processes of DMUs are ignored and the exogenous inputs consumed and final outputs produced by the DMUs are the only consideration for efficiency evaluation. On the other hand, network DEA (NDEA) attempts to formulate the internal operations of the evaluated DMU and thus intermediate products (which are outputs coming from divisions (sub-processes) and inputs utilized by others) are explicitly taken into account. In other words, NDEA intends to open the black box so as to provide adequate detail to management and can provide detailed information on the efficiency of divisions (or sub-DMUs) at the assessed DMU as well as its efficiency status. NDEA can be thought of as a generalization of standard DEA.

DEA researchers developed various NDEA models for evaluating the efficiency of DMUs [11,12,13,20,21,22,26,28,31,33] and other researchers focused on the efficiency performance of DMUs which have internal series (e.g., two-stage or three-stage) structures [32,23,27,16,9,17,18,1]. A review of the NDEA models can be found in Kao [21]. The management of the DMU often would like to know the sources of inefficiency within it, but some of the existing network DEA methods do not fully provide information on the DMU's overall efficiency status that is consistent with Pareto–Koopmans efficiency with the full consideration of internal flows or intermediate products. Obviously, adoption of Pareto–Koopmans efficiency stems from the possibility principle or free disposal hull which is for example given in the A3 postulate of Cooper et al. [10]. In NDEA Tone and Tsutsui [33] used the possibility principle for only exogenous inputs and final outputs similar to standard DEA which doesn't have intermediate products.

In the DEA literature, there are two efficiency notions: weak efficiency and Pareto–Koopmans efficiency. The Farrell–Debreu measure is calculated based on the weakly efficient frontier and hence possibly existing nonzero slacks are ignored in its efficiency measurement. By contrast, the original model by Charnes et al. [3] is developed with the intension of incorporating the notion of full efficiency or Pareto–Koopmans efficiency. Charnes, Cooper and their associates have incorporated this efficiency notion with the use of the non-Archimedean infinitesimal. In a black-box setting, a DMU is Pareto–Koopmans efficient if and only if it is impossible to make an improvement in the utilization of any input or output without worsening some of the other inputs and/or outputs. Hence, Charnes, Cooper and their associates relate the CCR model to the notion of Pareto–Koopmans efficiency. See Charnes and Cooper [6,7], Charnes et al. [8] and Cooper et al. [10] for detailed discussion of the difference between the two notions. The additive model, Russell models and slacks-based models are alternative methods to incorporate Pareto–Koopmans efficiency.

Another motivation for using Pareto–Koopmans efficiency is that we can provide a criterion to improve overall system efficiency in NDEA – we will show how the notion can be utilized to obtain an efficient target based on this criterion.

The present study analyzes NDEA with respect to Pareto–Koopmans efficiency for the situation where intermediate products are not supplied or demanded outside the assessed system or DMU. Moreover, we assume that all intermediate products are desirable. The studies that are explicitly based on Pareto–Koopmans efficiency with respect

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to the evaluated DMU and its divisions, include Lewis and Sexton [26] and Fukuyama and Mirdehghan [15]. In a two-stage problem where stage 1's outputs are the only inputs to stage 2, Lewis and Sexton [26, p. 1374] stated that a necessary condition for a DMU to be overall efficient is that each division is fully efficient, but efficiency in all sub-DMUs or divisions is not sufficient for overall efficiency of the DMU. Their definition of overall system efficiency is based on Pareto–Koopmans efficiency, which is the situation where it is not possible for a DMU to improve any exogenous input, final output or intermediate product without worsening some other exogenous inputs, final outputs or intermediate products. In a general NDEA setting where each division can have the three types of production variables, Fukuyama and Mirdehghan [15] showed how to identify the overall system efficiency status of DMUs for the fixed link[1] formulation where each observed intermediate product is restricted between the intermediate output of one division and the intermediate input of another. For the free link formulation, however, a straightforward application of Fukuyama and Mirdehghan's method (2012) does not always identify the overall efficiency status of the assessed DMU which consists of divisions or sub-DMUs. Therefore, the purpose of the present study is not only to implement the notion of Pareto–Koopmans efficiency to the determination of a DMU's overall efficiency status but also to show how to gauge divisional efficiencies within the free link network framework[2]. For this purpose, we provide a novel necessary and sufficient condition for a DMU to be Pareto–Koopmans efficient in general NDEA. Our two-phase approach[3] is based on the enhance Russell graph model of Pastor et al. [30]. We adopt this model because: (i) the original NDEA contributions have been made based on slacks-based models [22,33,34]; and (ii) the original efficiency measurement framework for dealing with positive slacks, or equivalently asymmetric scaling factors, is developed under input orientation [14].

The organization of the paper is as follows. Section 2 provides two motivating examples as well as the basics and then develops a new network efficiency measurement framework. Section 3 makes comparisons with some other network models using two artificial numerical examples, and then shows the use of some existing approaches can lead to a non Pareto–Koopmans efficient solution. In Section 4 we apply the proposed framework to the data of 27 Taiwanese banks as a real-life application. The last section concludes with several remarks. All the proofs of the theorems are relegated to Appendix A.

2. Dominance, motivating examples and network DEA

2.1. Mathematical dominance and Pareto–Koopmans efficiency

The basic definition of efficiency in multiple criteria decision making, particularly in DEA, is provided from a mathematical dominance (Pareto–Koopmans efficiency) perspective. However, Pareto–Koopmans efficiency is not necessarily utilized in NDEA. In actual fact, there are many NDEA studies, whose results are inconsistent with Pareto–Koopmans efficiency. That is, the evaluated DMU, rated as efficient in these NDEA models, can be dominated by another observed DMU, in which situation Pareto–Koopmans efficiency is violated.

In order to deal with this problem (the violation of Pareto–Koopmans efficiency) we suggest a two-phase approach based on three dominance notions – our suggested definition of overall system efficiency in NDEA exactly corresponds to the case where the NR measure is one (i.e., phase-1 efficient) and all divisions are efficient.

We start with three mathematical dominance notions with respect to pair-wise comparisons: (i) full product vector dominance, (ii) sub-vector dominance, and (iii) dominance at the division level. DMU_a, consisting of divisions or sub-DMUs, is said to fully dominate DMU_b if the full product vector of DMU_a dominates the corresponding product vector of DMU_b, where the full vector comprises the total amounts of not only exogenous inputs and final outputs but also intermediate products. DMU_a is said to sub-vector dominate DMU_b if $DMU_a{'}s$ sub-vector dominates the corresponding sub-vector of DMU_b, where the sub-vector consists of only exogenous inputs and final outputs (without intermediate products). Therefore, the first definition of dominance considers all division's intermediate products, whereas the second does not. The third definition only deals with dominance at a division level of the evaluated DMU.

Here, we make pair-wise vector comparisons between different DMUs when internal flows exist. The three definitions are utilized to determine the Pareto–Koopmans efficiency status of the evaluated DMU. The three dominance notions are formally presented in the next section.

The adoption of vector dominance allows us to distinguish three kinds of efficiency: (a) Pareto–Koopmans efficiency, (b) sub-vector efficiency, and (c) divisional efficiency. Pareto–Koopmans efficiency at the evaluated DMU is determined by full-vector dominance. By contrast, the sub-vector efficiency status at the DMU is identified by sub-vector dominance. Sub-vector efficiency is often used to define efficiency in NDEA. Note that Pareto–Koopmans efficiency implies sub-vector efficiency but not the other way around. See Lewis and Sexton [26, p. 1374] and Castilli et al. [2, p. 222] on this point.

2.2. Motivating examples

In this subsection, we provide two motivating examples from a model building point of view. Consider Fig. 1 that consists of two DMUs, each of which has two divisions, and the DMUs employ one exogenous input, two intermediate products and one final output. The amounts of exogenous inputs and final outputs of the two DMUs are the same and the only difference between the two DMUs is the amount of intermediate products. Division 1 of DMU_2 produces a more amount of intermediate product than division 1 of DMU_1, even though the two DMUs consume the same amount of exogenous input and produce the same amount of final output. Based on the notion of dominance at the division level defined in the previous sub section, we conclude that Division 1 of DMU_2 and Division 2 of DMU_1 are efficient and Division 1 of DMU_1 and Division 2 of DMU_2 are inefficient. Now consider the two-stage constant-returns-to-scale network CCR (NCCR) model due to Kao and Hwang [23]. The NCCR model is a two-stage network model, in which the first division's outputs are the only inputs to the second stage. Clearly, the application of the NCCR model to Example 1 leads to the situation contradictory to Pareto–Koopmans efficiency because Divisions 1 and 2 are considered efficient for both DMUs.

Next, we show that the NCCR model does not find an efficiency score uniquely using Example 2 depicted in Fig. 2. This result is of great importance because multiple solutions are inconsistent with Pareto–Koopmans efficiency. The efficiency of DMU_2 using the NCCR model is obtained by solving the following linear program:

$$\begin{aligned} & Max \quad 3u \\ & s.t. \quad 2v = 1 \\ & 2w_1 + w_2 - v \le 0 \qquad (1.1) \\ & 2u - 2w_1 - w_2 \le 0 \qquad (1.2) \\ & w_1 + 2w_2 - 2v \le 0 \qquad (1.3) \\ & 3u - w_1 - 2w_2 \le 0 \qquad (1.4) \\ & u,\ v,\ w_1,\ w_2 \ge \varepsilon \end{aligned} \tag{1}$$

[1] The term "fixed link" is used by Tone and Tsutsui [33] to deal with the situation where the linking activities of a DMU are fixed and hence the intermediate products are discretionary (beyond the control of management). The present study develops an alternative general network framework that deals with Pareto–Koopmans efficiency. For a more discussion on Tone and Tsutsui's [33] fixed link case, see Fukuyama and Mirdehghan [15].

[2] This research focuses on a framework in which inputs and outputs are not shared. See Castelli et al. [2] for a framework of shared inputs and outputs across divisions.

[3] In this paper the terms "model" and "measure" mean an efficiency measurement mathematical problem and its optimized objective function value, respectively. The two-phase "approach" is our proposed two-phase method, in which the Pareto–Koopmans efficiency statuses of DMUs are identified by solving two models in two phases.