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Multi-output profit efficiency and directional distance functions ☆

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ABSTRACT

We extend a recently developed DEA methodology for cost efficiency analysis towards profit efficiency settings. This establishes a novel DEA toolkit for profit efficiency assessments in situations with multiple inputs and multiple outputs. A distinguishing feature of our methodology is that it assumes output-specific production technologies. In addition, the methodology accounts for the use of joint inputs, and explicitly includes information on the allocation of inputs to individual outputs. We also establish a dual relationship between our multi-output profit inefficiency measure and a technical inefficiency measure that takes the form of a multi-output directional distance function. Finally, we demonstrate the empirical usefulness of our methodology by an empirical application to a large service company.

1. Introduction

Production processes that generate multiple outputs are typically characterized by jointly used inputs, i.e. inputs that simultaneously benefit different outputs. These joint inputs give rise to economies of scope, which actually form a prime economic motivation for Decision Making Units (DMUs) to produce more than one output. In the current paper, we establish a methodology for multi-output profit efficiency evaluation that explicitly accounts for jointly used inputs. In particular, our methodology distinguishes between joint inputs and inputs that are allocated to specific outputs.

DEA analysis of multi-output production: The method that we develop fits within the popular Data Envelopment Analysis (DEA, after [8]) approach to productive efficiency measurement. This DEA approach is intrinsically nonparametric, which means that it does not require a parametric/functional specification of the (typically unknown) production technology. It "lets the data speak for themselves" by solely using technological information that is directly revealed by the observed production units. It then reconstructs the production possibility sets by (only) assuming standard production axioms (such as monotonicity and convexity).[1] A DMU's efficiency is measured as the distance of the corresponding input-output combination to the efficient frontier of this empirical production set. Typically, a DMU's efficiency can be computed by simple linear programming. Its nonparametric nature and its easy computation largely explain DEA's widespread use as an analytical research instrument and decision-support tool.

Recently, Cherchye et al. [10,9] introduced a novel DEA methodology to analyze cost efficiency in multi-output settings. The methodology assumes output-specific production technologies, accounts for joint inputs in the production process, and incorporates specific information on how inputs are allocated to individual outputs. As such they provide a formal modeling of the economies of scope that characterize the multi-output production process.[2] These authors have also shown that their cost efficiency measure evaluated at shadow prices is dually equivalent to a specific multi-output version of the [21–30] measure of (radial) input efficiency. This is an attractive feature, as DEA practitioners often use this Debreu–Farrell measure for evaluating the technical efficiency of a DMU's input use (when assuming a fixed output).

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[1] See [28,20,19,31,18] for extensive reviews of DEA. From an economic perspective, DEA itself is rooted in the structural approach to modeling efficient production behavior that was initiated by Afriat [1], Hanoch and Rothschild [34], Diewert and Parkan [23] and Varian [43]. Given the explicit economic motivation of our following analysis, our contribution also fits in this tradition of structural efficiency analysis.

[2] See also [42,40] for related work on modeling economies of scope in a DEA context.

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At this point, we remark that the methodology of [10,9] is closely related to several existing approaches in the DEA literature. Firstly, there is a clear connection with network DEA (see [26,29]). The literature on network DEA also makes use of what we define further as output-specific inputs. However, to the best of our knowledge, it abstracts from the possibility of jointly used inputs. These joint inputs play an important role in our approach because they define the interdependencies between the production processes associated with different outputs and, as a result, they characterize the economies of scope that underlie the observed production processes. Secondly, Salerian and Chan [41] and Despic et al. [22] present two alternative methods to model inputs that contribute to some outputs but not to others. As such, these models can actually be interpreted as special cases of our model with joint inputs, but without having output-specific inputs.

Summarizing, these alternative approaches have in common that they try to enhance the realism of the efficiency evaluation exercise by integrating information on the internal production structure. In a sense, we provide a unifying framework that integrates these existing approaches. This framework should be particularly attractive to empirical researchers who are familiar with standard DEA techniques and interested in the analysis of multi-output production characterized by joint inputs.

Profit efficiency analysis: The current paper extends this methodology for multi-output efficiency assessments to profit efficiency settings, which makes our paper fit in the extensive literature that studies profit efficiency and its extensions in a DEA context.[3] In many practical settings, profit efficiency is considered to be the best suited criterion for evaluating the performance of productive activities. In addition, by its very definition cost efficiency is a necessary condition for profit efficiency. Profit efficiency evaluations are generally more stringent than cost efficiency evaluations. As a result, they can signal additional sources of inefficiency and, thus, potential performance improvements. In this respect, as we will indicate, an appealing feature of our multi-output approach is that it also allows us to allocate a DMU's aggregate profit inefficiency to individual outputs. This helps to better identify specific output production processes where substantial profit efficiency gains are possible, which can usefully assist DMU managers to direct their performance improvement actions in an effective way (i.e. primarily towards outputs that are characterized by considerable inefficiency).

In developing our profit efficiency methodology, we also start from output-specific technologies and distinguish between joint inputs and output-specific inputs in the process of multi-output production. Next, we will show that our profit inefficiency measure under shadow prices has a dual representation as a directional distance function. We believe this is an interesting property, as directional distance functions have become increasingly popular as a technical inefficiency measure that simultaneously includes outputs produced and inputs used. Basically, this duality result extends the one of [7] towards our specific multi-output setting. A particular feature of our analysis here is that we explicitly account for output-specific technologies with jointly used inputs in establishing the duality relationship.

Outline: The rest of the paper is structured as follows. Section 2 introduces some necessary notation and terminology. Section 3 introduces our method for multi-output profit inefficiency measurement. Section 4 establishes the dual representation of our profit inefficiency measure as a directional distance function. Section 5 shows the practical usefulness of our method through an application to a large service company. Section 6 concludes.

[3] See, for example, [14,35,38,36,39,4] for recent contributions.

2. Preliminaries

The distinction between inputs and outputs becomes less relevant in profit efficiency analysis. Therefore, to simplify notation it will often be convenient to work with "netputs" in our following exposition. As we will explain, netput vectors simultaneously capture inputs used (as negative components) and outputs produced (as positive components). We will define this netput concept for our specific setting with joint and output-specific inputs. In turn, this will allow us to introduce our notion of output-specific technologies and, correspondingly, our particular concept of multi-output profit.

Netputs and multi-output technologies: We consider a production technology that uses N inputs to produce M outputs, which we represent by the vectors $\mathbf{X} = (x^1, \ldots, x^N)' \in \mathbb{R}^N_+$ and $\mathbf{Y} = (y^1, \ldots, y^M)' \in \mathbb{R}^M_+$, respectively. Our method distinguishes between joint and output-specific inputs.

- *Output-specific* inputs are allocated to individual outputs m, i.e. they specifically benefit the production process of (only) the m-th output. In our formal analysis, we will use $\alpha_k^m \in [0,1]$ (with $\sum_{m=1}^{M} \alpha_k^m = 1$) to represent the fraction of the k-th output-specific input quantity that is allocated to output m.
- *Joint* inputs are not allocated to specific outputs but are simultaneously used in the production process of *all* the outputs. Clearly, these joint inputs generate interdependencies between the production processes of different outputs.[4]

In the following we will assume that the allocation parameters α_k^m are observed. We believe that in many instances this is not a strong assumption, since large firms nowadays often use cost systems that explicitly allocate inputs/costs to outputs (e.g. Activity Based Costing (ABC)) to support various strategic and operational decisions. These cost systems can be used to define the α_k^m (see, for example, our own empirical application in Section 5). Nevertheless, if this information is not available to the empirical analyst, we can make use of alternative approaches that are not based on observing this information, but try to reconstruct the decomposition (over outputs) of the output-specific inputs in DEA analysis itself.[5] Cherchye et al. [10] provide a discussion on how to integrate these techniques in the approach to multi-output efficiency analysis that we present here. These authors' discussion focused on a cost efficiency setting, but it readily extends to the profit efficiency setting that we consider in the current paper.

We will represent the allocation of inputs to outputs by means of a vector $\mathbf{A}^m \in \mathbb{R}^N_+$ for each output m, for which the entries are defined as (with $\alpha_k^m \in [0,1]$ and $\sum_{m=1}^{M} \alpha_k^m = 1$)

$$(\mathbf{A}^m)_k = \begin{cases} 1 & \text{if input } k \text{ is joint and used to produce output } m, \\ \alpha_k^m & \text{if input } k \text{ is output} - \text{specific and used to produce output } m, \\ 0 & \text{if input } k \text{is not used to produce output } m. \end{cases}$$

Then, each vector $\mathbf{A}^m$ defines the input vector $\mathbf{X}^m = \mathbf{A}^m \odot \mathbf{X}$, which thus contains the input quantities used in the production process of output m.[6]

As indicated above, we can often simplify our notation by working with netputs, which simultaneously stand for outputs and inputs. Specifically, we use $\mathbf{Z} = \begin{bmatrix} \mathbf{Y} \\ -\mathbf{X} \end{bmatrix} \in \mathbb{R}^{M+N}$ to denote the

[4] See [13] for the introduction of sub-joint inputs. These inputs play a similar role as joint inputs, but only for a *subset* of (instead of all) outputs. It is straightforward to include this third type of inputs in our methodology, but for the ease of the exposition we abstract from this in the current paper.

[5] See, for example, [17,16,3,37,44,24].

[6] The symbol $\odot$ stands for the Hadamard (or element-by-element) product.