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Exact approaches for the pickup and delivery problem with loading cost [☆]

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ABSTRACT

In this paper, we propose a branch-and-cut algorithm and a branch-and-price algorithm to solve the pickup and delivery problem with loading cost (PDPLC), which is a new problem derived from the classic pickup and delivery problem (PDP) by considering the loading cost in the objective function. Applications of the PDPLC arise in healthcare transportation where the objective function is customer-centric or service-based. In the branch-and-price algorithm, we devise an ad hoc label-setting algorithm to solve the pricing problem and employ the bounded bidirectional search strategy to accelerate the label-setting algorithm. The proposed algorithms were tested on a set of instances generated by a common data generator in the literature. The computational results showed that the branch-and-price algorithm outperformed the branch-and-cut algorithm by a large margin, and can solve instances with 40 requests to optimality in a reasonable time frame.

1. Introduction

The pickup and delivery problem with loading cost (PDPLC) consists in deciding a set of minimal cost routes which comply with vehicles' capacity constraints and maximal duration constraints, to satisfy a set of transportation requests. Let $G=(V,A)$ be the complete graph where the PDPLC is defined. $V=\{0,\dots,2n+1\}$ is the node set, where nodes 0 and $2n+1$ are two duplicates of the depot, and the subsets $P=\{1,\dots,n\}$ and $D=\{n+1,\dots,2n\}$ are the set of pickup nodes and the set of delivery nodes, respectively. $A=\{(i,j)\,|\,i,j\in V, i\neq j\}$ is the arc set. Each transportation request involves transporting goods from a pickup node $i\in P$ to the corresponding delivery node $n+i\in D$. For convenience, we also use P to refer to the set of transportation requests. Each node $i\in V$ has a demand q_i, which satisfies $q_0=q_{2n+1}=0$, $q_i=-q_{n+i}$ and $q_i>0$ for all $i\in P$. And each arc $(i,j)\in A$ has a travel distance $d_{i,j}$. The travel distance matrix $d_{i,j}$ $((i,j)\in A)$ is assumed to satisfy the triangle inequality, i.e., $d_{i,j}\leq d_{i,k}+d_{k,j}$ for any arcs $(i,j),(i,k),(k,j)\in A$. Let K denote the set of identical vehicles. Each vehicle has a capacity Q and a maximal duration T. Let D_k denote the travel distance of vehicle $k\in K$ and D_i denote the travel distance of request $i\in P$. Then the objective of the PDPLC is defined to be $\sum_{k\in K}D_k+\lambda\sum_{i\in P}q_iD_i$, where $\sum_{k\in K}D_k$ is referred to as the *routing cost*, $\sum_{i\in P}q_iD_i$ is referred to as the *loading cost*, and λ is a positive coefficient coordinating the routing cost and the loading cost. From a mathematical point of view, the PDPLC can be viewed as a generalization of the classic *pickup and delivery problem* with loading cost consideration in the objective.

The PDPLC is motivated by the observation of the vehicle scheduling in the Non-Emergency Ambulance Transfer Service (NEATS), a free service offered by the Hong Kong Hospital Authority (HKHA) which provides non-emergency transportation to/from medical institutions for the elderly or patients [24,43]. The number of transportation requests faced by the NEATS varies day by day, from dozens to more than a hundred. The actual scheduling in the NEATS involves different various resources, including vehicles like ambulances and mini buses, stretchers, wheel chairs, oxygen cylinders as well as the most complicated manpower like drivers and assistances. Currently, the vehicles in the NEAT are manually scheduled by several experienced controllers at a center. One of the major drawbacks of manually scheduling is that there is a tendency for the controllers to minimize travel distance of vehicles, while neglecting the service quality, which is mainly measured by the passengers' riding time. Actually, nowadays with the increasing number of requests, the HKHA is facing more and more criticisms from the public, due to the deteriorating service quality. If the controllers are forced to minimize the passengers' riding time, then they will tend to assign a vehicle to transport a user from her/his

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pickup point to her/his delivery point directly, which may incur a very high travel distance of vehicles, or worse, lead to the inability to serve all of the requests. Therefore, it is crucial for a decision support system applied for the NEAT to achieve a very good tradeoff between operational cost and service quality. In Hong Kong, many elderly people live in public housing provided by the government. Every day, several people from the same public housing division may require the NEATS to transfer them to the same local hospital. Therefore, the number of users in a request of NEATS can be treated as the quantity of goods in a request of the PDPLC and the total riding time of users is equal to the loading cost of the PDPLC. Another interesting application of the PDPLC arises in the O2O (online to offline) food take-out E-commerce. Today, many major O2O food E-commerce enterprises in China have set up their own delivery teams who are in charge of delivering food take-out from restaurants to customers. One of the major advantages to set up one's own delivery team is that it enables the E-commerce enterprises to provide standard and high quality service. One of the major criteria to measure the service quality of O2O food take-out delivery service is the time of food on the road, because this criterion heavily influences the tastes of food, especially for many Chinese foods, and hence should be considered in the objective when the delivery teams are scheduled.

The pickup and delivery problem (PDP) has been extensively studied in the literature for decades. The objective of the classic PDP is either to minimize the total travel distance of vehicles, or to first minimize the number of vehicles and then the total travel distance of vehicles. Many variants of the PDP have been proposed by researchers, such as the PDP with time windows [31,21,37], the PDP with transfers [7,28], the dynamic PDP [3,30], etc. For a detailed survey of the PDP, see Berbeglia et al. [2]. Most of the branch-and-price algorithms for the PDP in the literature have been proposed to solve the PDP with time windows (PDPTW), which can be formulated into a set-partitioning model as required by branch-and-price algorithms. In set-partitioning models for the PDPTW, each decision variable corresponds to a route. Therefore, the number of decision variables is huge because there exists a large number of feasible routes. Dumas et al. [13] first proposed a set-partitioning model for the PDPTW and devised a branch-and-price algorithm to solve this problem. Because the number of decision variables in this set-partitioning models was exponential, its linear programming (LP) relaxation was solved by column generation, in which the pricing problem was a shortest path problem with pickup, delivery, time window and capacity constraints. The pricing problem was solved by a forward label-setting algorithm. Savelsbergh and Sol [39] proposed a similar branch-and-price algorithm which applied several heuristics to solve the pricing problem and a sophisticated column management scheme to achieve an optimal balance between solution speed and solution quality. A branch-and-price-and-cut algorithm which employed several sets of valid inequalities to strengthen the set-partitioning model and a new label-setting algorithm to solve the pricing problem, was proposed by Ropke and Cordeau [35] to solve the PDPTW.

In recent years, there has been an increasing amount of literature on the vehicle routing problem with loading cost (VRPLC). Tang et al. [42] first introduced the loading cost structure into the capacitated vehicle routing problem (CVRP) and designed a scatter search algorithm for the resultant CVRP with loading cost. Kuo and Wang [20] addressed the multi-depot vehicle routing problem with loading cost (MDVRPLC) and proposed a variable neighborhood search algorithm to solve it. The split-delivery weighted vehicle routing problem, which is also a variant of VRPLC, was proposed by Tang et al. [41] and solved by a max–min of any system. Compared with the classic vehicle routing problem, in which the cost is independent of the load on a vehicle, the VRPLC can model the reality more accurately in certain contexts. For example, in the transportation of perishable goods (e.g. fruits, vegetables, and meat), the more time these goods spend in the vehicle, the more value they lose due to decay. The lost value of these perishable goods can be roughly measured by the product of their quantities and travel distances. A similar example is the transportation of dangerous goods (e.g., toxic chemical products and nuclear materials), which have a very high risk when they are being transported on the road. This kind of risk, which is proportional to the goods' quantities and travel distances, should be considered in the cost of their transportation. Another interesting application of the VRPLC can be found in Chinese expressways where the amount of money paid by drivers to the expressway's companies depends on both the travel distance and the weight of the vehicles, which is referred as the *toll-by-weight* scheme. By 2013, most of the provinces in China had adopted the toll-by-weight scheme and many logistics companies in China had to redesign their scheduling plans for vehicles to save cost under this new tolling scheme. Zhang et al. [44] first studied the traveling salesman problem in the context of the toll-by-weight scheme and proposed a branch-and-bound algorithm to solve the resultant problem.

A problem similar to the PDPLC in the literature is the dial-a-ride problem (DARP) [40,18,6], which is a special PDP arising in the door-to-door transportation of the elderly or disabled people. Because human beings instead of freights are transported in the DARP, some customer-centric or service-based cost measurements, such as customer waiting time, customer riding time, and the difference between the actual delivery time and the desired delivery time, are incorporated into the objective function. Therefore, when a DARP aims to minimize the total travel distance of vehicles and the total riding time of customers simultaneously, it shares some similarities with the PDPLC. However, their major difference is that the total riding time in the DARP is equal to the total travel distance of customers and independent of requests' demands; whereas, the loading cost in the PDPLC is the weighted sum of goods' (customers') travel distances. We refer the readers to Cordeau and Laporte [5], Melachrinoudis et al. [29] for a detailed survey on the DARP. Exact algorithms for the DARP include dynamic programming algorithms for the single vehicle DARP [32,12], branch-and-cut algorithms [4] and branch-and-price algorithms [16].

In this paper, we conduct a detail computational study on the PDPLC with a branch-and-cut algorithm and a branch-and-price algorithm, which are two classes of leading exact algorithms to solve many combinatorial optimization problem in recent years [22,27,23,9]. The design of the branch-and-cut algorithm is based on a compact arc-flow model and seven families of valid inequalities, while the branch-and-price algorithm relies on the strong lower bound provided by the linear programming relaxation (LP) of a set-partitioning model with an exponential number of decision variables. Although the valid inequalities can also be applied to the set-partitioning model and can be used to strengthen the set-partitioning model in theory, our preliminary computational results show that these inequalities have little influence on the set-partitioning model in practice, and hence they are ignored in the set-partitioning model. The LP relaxation of the set-partitioning modeling is solved by column generation, which has an elementary shortest path problem with capacity, maximal duration, pairing and precedence constraints and loading cost as the pricing problem. An ad hoc label-setting algorithm, accelerated by the bounded bidirectional search, is proposed to solve the constrained shortest path problem with loading cost. Both the branch-and-cut algorithm and the branch-and-price algorithm were tested on a set of instances which were generated by a common data generator in the literature, with different values assigned to the maximum duration T and the objective coefficient λ. Our computational results show that the branch-and-price algorithm outperformed the branch-and-cut