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Two-phase algorithm for the lot-sizing problem with backlogging for stepwise transportation cost without speculative motives ☆

Hark-Chin Hwang [a], Jangha Kang [b,*]

[a] School of Management, Kyung Hee University, 26 Kyunghee-daero, Dongdaemun-gu, Seoul 130-701, Republic of Korea
[b] Department of Industrial Engineering, Chosun University, 309 Pilmun-daero, Dong-Gu, Gwangju 501-759, Republic of Korea

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ABSTRACT

In this paper we consider the lot-sizing problem with backlogging under stepwise transportation costs. Inventory is carried over or backlogged in a trade-off with costs for production setup and transportation. Specifically, inventory is the main source for consolidating demand over periods to increase the chance of Full-Truck-Load (FTL) delivery. We assume that there are no speculative motives in production, which yields an important property for Less-Than-Load (LTL) delivery that the LTL cargo does not contain any unit carried over from the previous period or backlogged for the next period. We solve the problem in two phases. In phase one, we use a geometric technique to preprocess necessary functional values for FTL delivery. In phase two, we provide a residual zoning algorithm, involving not only FTL delivery but also LTL delivery, to obtain an optimal solution. The computational complexity is shown to be $O(T^2 \log T)$ where T is the length of the planning horizon.

1. Introduction

In this paper we consider a lot-sizing problem for deterministic multi-period production and transportation planning, which we call the lot-sizing problem with production and transportation (LSPT). Item units, which are produced with setup and per-unit costs, are contained in the *cargo* that is transported by truck or railroad. Transportation cost is charged by cargo, which leads to a *stepwise transportation cost structure*.

To provide a context for the LSPT, we consider a division of a global company with several production sites and distribution centers around the world. The division is in charge of a facility producing a single item and a distribution center facing the deterministic demand of a local region. The consideration of production, inventory and transportation is crucial for effective decision-making. The company manages production and inventory internally; on behalf of the company, independent carriers transport items from the production facility to the warehouse in the distribution center. Therefore, because the per-unit delivery cost of cargo depends on the independent carrier chosen in a given period, this cost may vary over periods.

In traditional inventory management literature, inventory—including backlogged demand—accrues in a trade-off with production setup cost. In this study, inventory is also used for the source of demand consolidation over periods, whereby we can increase the amount of item units in a single delivery, reducing transportation costs [10]. As a consequence, we maintain inventory in a trade-off with the production setup and cargo delivery costs. There is no other reason for maintaining inventory, i.e., no speculative motive prevalent in the oil market, for example, which fosters keeping oil on hand in anticipation of rising prices.

Lippman [12] introduced the lot-sizing problem with multiple setups to model the stepwise cost, thereby incurring a setup cost each time a new cargo is created. The research extending Lippman's model includes Pochet and Wolsey [13] and Van Vyve [17]. In addition to the stepwise cost, Van Vyve [17] also considered the maximum bound on the number of cargos that can be used in each period. When only one cargo is allowed in each period, the problem reduces to the classical capacitated lot-sizing problem [6,4,16]. The research in this direction is reviewed by Karimi et al. [7] for the single and multi-item capacitated problems and by Robinson et al. [14] for the multi-item capacitated problem involving joint setups. Efficient heuristic approaches are proposed by Chen [3] for a generalized multi-level capacitated lot-sizing problems including setup carry-overs.

To deal with not only the stepwise transportation cost but also the setup cost in production, Lee [9] first proposed the LSPT problem and Lee et al. [10] solved the problem with backlogging

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* Corresponding author. Tel.: +82 62 230 7126.
E-mail addresses: hchwang@khu.ac.kr (H.-C. Hwang), janghakang@chosun.ac.kr (J. Kang).

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and outbound transportation. Li et al. [11] and Hwang [8] presented improved algorithms for the LSPT problem that allows backlogging. Recently, Akbalik and Rapine [2] solved the LSPT problem with the bound on the number of cargos in the case where backlogging is not allowed.

The LSPT problem in this paper assumes production cost without speculative motives, a *nonspeculative cost structure*. (Section 2.1 gives the precise definition for the nonspeculative cost structure.) The objective of this paper is to provide an $O(T^2 \log T)$ algorithm for the LSPT problem with the nonspeculative cost structure improving upon the $O(T^3)$ algorithm in Hwang [8], where T is the length of the planning horizon.

Algorithms for lot-sizing problems have been continuously improved since the introduction of the uncapacitated lot-sizing problem [19]. Important improvements were made by Federgruen and Tzur [5], Wagelmans et al. [18] and Aggarwal and Park [1]. One interesting improvement method is the geometric technique, which constructs and updates at each iteration the envelope (convex hull) of points in a two-dimensional space of period and cost; and at the end of the iteration, the extreme points of the convex hull yield a production plan. Similar to the envelope of points approach, another equivalent technique using the envelope of lines (linear functions) is formalized in van Hoesel et al. [15].

Most lot-sizing problems involving capacity are solved first by decomposition into subproblems, and then determining the optimal solution. We refer to the sequential steps as *phases*. That is, in phase one, we obtain the minimum values of the subproblems; in phase two, we compute an optimal solution for the entire problem using the values of the subproblems. For efficiency in solving the subproblems, the geometric technique of linear functions will be applied in phase one. Although this geometric technique is quite useful for subproblems, it seems less so for solving the entire problem. To obtain the improved $O(T^2 \log T)$ algorithm, we propose a new technique called the *residual zoning algorithm* in phase two.

In the next section, we present the notation and problem formulations with basic optimality properties. Section 3 introduces the value functions and overall approach for the optimal algorithm. In Section 4, we describe phase one, which is to obtain functional values using the geometric technique. Phase two for our main algorithm with the residual zoning procedure is presented in Section 5. In Section 6, we will review the algorithm and offer the managerial insights behind its optimality properties, and future possible extensions.

2. Preliminaries

2.1. Problem formulation

As in the previous section, we assume a division of a global company in charge of a production facility and a distribution center, which faces demand d_t over periods $t = 1, \ldots, T$. Produced units in the facility are delivered via truck or railroad to the distribution center in cargo of capacity C, the maximum amount of product units that can be contained. For expositional simplicity, we assume that the demand $d_1, \ldots, d_T$ and the capacity C are integral values. It should not be difficult, however, to see that the algorithm could be applied to non-integral values as well. The production cost function in period t consists of the fixed setup cost K_t and the per-unit cost p_t. As transportation is served by independent carriers, the transportation cost can vary over time: each delivery of a cargo costs W_t in period t. The leftover items in period t incur a holding cost h_t per unit and the backlogged items are imputed with backlogging cost b_t per unit.

A production and transportation plan is specified by x_t and I_t for $t = 1, \ldots, T$ where x_t denotes the production and transportation level and I_t denotes the inventory level in period t. Let $I_t = I_t^+ - I_t^-$ where $I_t^+ \geq 0$ denotes the carried-over level and $I_t^- \geq 0$ the backlogged level at the end of period t. From the definitions of I_t^+ and I_t^-, it is obvious that $I_t^+ \cdot I_t^- = 0$. (Rigorous reasons for this can be given from Proposition 1.) We use y_t to indicate whether or not production occurs in period t; it takes the value 1 if production occurs, and 0 otherwise.

Regarding the cost associated with cargo capacity, we use $\lceil x \rceil$ and $\lfloor x \rfloor$ to denote the minimum integer no less than x and the maximum integer no greater than x, respectively. The production and transportation cost for $x_t \geq 0$ units in period t is then given as follows:

$$P_t(x_t) = p_t x_t + \lceil x_t / C \rceil W_t.$$

The production cost function in this study is assumed *nonspeculative* which means that

$$p_t - b_t \leq p_{t+1} \leq p_t + h_t \quad \text{for } t = 1, 2, \ldots, T-1. \tag{1}$$

Under the condition (1), we see that the unit cost of an item produced in the present period t is no larger than that of an item carried-over ($p_t \leq p_{t-1} + h_{t-1}$) and that of an item backlogged ($p_t \leq p_{t+1} + b_t$). Therefore, with the per-unit production and inventory costs satisfying the nonspeculative condition, we have no reason to hold stock and to backlog demand; the only reason that we keep stock is in a trade-off with (high) setup cost or transportation cost.

For facilitating arguments throughout the paper, we provide useful terms especially for handling cumulative quantities:

- $d_{s,t}$ denotes the sum of demands $d_s, d_{s+1}, \ldots, d_t$, i.e., $d_{s,t} = d_s + d_{s+1} + \cdots + d_t$. In an analogous way, we define $x_{s,t}$ to denote the sum of productions during $[s, t]$, i.e., $x_{s,t} = x_s + x_{s+1} + \cdots + x_t$.
- $b_{s,t}$ and $h_{s,t}$ denote the sum of per-unit backlogging and holding costs in periods $s, s+1, \ldots, t$, respectively. That is, $b_{s,t} = b_s + b_{s+1} + \cdots + b_t$ and $h_{s,t} = h_s + h_{s+1} + \cdots + h_t$.

All values $d_{s,t}$, $b_{s,t}$ and $h_{s,t}$ are set to zero if $s > t$. For instance, we let $d_{s,t} = 0$ if $s > t$. With the notation developed, the problem is formulated as follows:

$$(\textbf{LSPT}) \quad \min \sum_{t=1}^{T} \left(K_t y_t + P_t(x_t) + h_t I_t^+ + b_t I_t^- \right) \tag{2a}$$

subject to

$$(I_{t-1}^+ - I_{t-1}^-) + x_t = d_t + (I_t^+ - I_t^-), \quad t = 1, \ldots, T \tag{2b}$$

$$I_0^+ = I_0^- = I_T^+ = I_T^- = 0, \tag{2c}$$

$$y_t \in \{0, 1\}, \quad t = 1, \ldots, T, \tag{2d}$$

$$x_t \geq 0, \quad I_t^+ \geq 0, \quad I_t^- \geq 0, \quad t = 1, \ldots, T. \tag{2e}$$

The objective is to find a production and transportation plan minimizing the setup, production and transportation and inventory costs as described in (2a). The over- or under-production of each period is balanced out by inventory (Eqs. (2b)). We assume that the inventory level at the start and at the end of the horizon is zero (Eqs. (2c)). With the assumption that d_t, $t = 1, \ldots, T$ and C are integers, we can always assume an integral production and transportation plan; i.e., x_t and I_t are all nonnegative integers (see Proposition 2 in Section 2.2).

If $x_t > 0$ we call period t a *production and transportation* period, or just a *production* period in short. Furthermore, if the amount of production in period t is a multiple of cargo capacity, we call it an *FTL* (*Full-Truck-Load*) production (and transportation) period. On