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On fuzzy multiattribute decision-making models and methods with incomplete preference information☆

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Abstract

In the paper by Li (*Fuzzy Sets and Systems* 106 (1999) 113), a minimum average weighted deviation method was investigated to determine the weights of attributes using incomplete preference information. It is shown in this paper that the method is flawed and can only be meaningful if it is revised. Two general ways are presented to revise it and are justified by using Lagrange multiplier method and Hessian matrix. Four specific revised models and approaches are investigated and examined through a numerical example. Different membership functions are suggested and utilized to normalize decision matrix so that a credible ranking order can be achieved.

Keywords: Multiattribute decision making; Weight estimation; Lagrange multiplier method; Hessian matrix

1. Introduction

In the paper [2], Li extended the method without preference information about the weights of attributes, which was first put forward by Wang and Fu [5], to the situation with incomplete preference information and presented two methods. One is an average weighted programming method and the other is a minimum average weighted deviation method. The former method is almost the same as the one proposed by Wang

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and Fu [5]; the latter method is a new one. However, as a weight estimation technique the new method is found flawed theoretically. This will be shown in the paper and revised models and methods are presented and justified instead.

The paper is organized as follows: Section 2 gives a brief description of the minimum average weighted deviation method. Section 3 provides a theoretical proof and shows why the method is flawed. Revised models and methods are presented and justified in Section 4. A numerical example is examined and illustrated in Section 5. The paper is concluded in Section 6.

2. The minimum average weighted deviation method for determining the weights of attributes

Suppose a decision problem has n alternatives $A_1, \ldots, A_n$ and m decision attributes $G_1, \ldots, G_m$. Each alternative is evaluated with respect to every attribute. All the values of attributes constitute a decision matrix denoted by $X = (x_{ij})_{n\times m}$. Generally speaking, decision attributes may be grouped into at most four groups $\Omega_1, \ldots, \Omega_4$, which represent benefit, cost, target and interval attributes, respectively [5]. So-called benefit attributes are those attributes for maximization, while cost attributes are those for minimization. Target attribute means that there exists a best value for such an attribute. The best value, however, does not mean the biggest or the smallest value. For example, it is a common sense that a constant voltage regulator should keep its output at its best level, say 220 voltage. Any voltage higher than or less than this figure is undesirable. Thus, the output of a constant voltage regulator should be considered as a target attribute. Target attributes can be well modeled by using triangular fuzzy numbers. So-called interval attribute means that there exists an interval for such an attribute and when its attribute value lies within this interval its utility is the best. For example, when choosing a leader from among a group of candidates, age is an important attribute. The selected leader can neither be too young nor too old. If a candidate is too young, he/she may not be experienced. If a candidate is too old, he/she is unlikely to be energetic. So, there exists a suitable interval for the age attribute in the decision problem. Such an age attribute may be seen as an interval attribute. Note that interval attributes do not mean their values are intervals. Their attribute values are still single values. Trapezoidal fuzzy numbers are particularly suitable for the characterization of interval attributes. When the interval of best values reduces to a single value, the interval attribute becomes a target attribute. So, target attributes are special cases of interval attributes.

Because of the incommensurability among attributes, the decision matrix $X = (x_{ij})_{n\times m}$ has to be normalized before conducting decision analysis. However, there are different ways that can be used to normalize the original decision matrix, which may lead to different decision results. Which way should be used depends mainly on the decision analyst. It would be better if the decision analyst can use several different normalization methods to conduct a sensitivity analysis. The most commonly used normalization methods are listed below [5], which utilize different fuzzy membership functions [6] to normalize different types of attributes, respectively:

$$z_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \quad i = 1, \ldots, n, \quad j \in \Omega_1, \tag{1}$$

$$z_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}}, \quad i = 1, \ldots, n, \quad j \in \Omega_2, \tag{2}$$