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Robustness analysis methodology for multi-objective combinatorial optimization problems and application to project selection ☆

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ABSTRACT

Multi-objective combinatorial optimization (MOCO) problems, apart from being notoriously difficult and complex to solve in reasonable computational time, they also exhibit high levels of instability in their results in case of uncertainty, which often deviate far from optimality. In this work we propose an integrated methodology to measure and analyze the robustness of MOCO problems, and more specifically multi-objective integer programming ones, given the imperfect knowledge of their parameters. We propose measures to assess the robustness of each specific Pareto optimal solution (POS), as well as the robustness of the entire Pareto set (PS) as a whole. The approach builds upon a synergy of Monte Carlo simulation and multi-objective optimization, using the augmented ε-constraint method to generate the exact PS for the MOCO problems under examination. The usability of the proposed framework is justified through the identification of the most robust areas of the Pareto front, and the characterization of every POS with a robustness index. This index indicates a degree of certainty that a specific POS sustains its efficiency. The proposed methodology communicates in an illustrative way the robustness information to managers/decision makers and provides them with an additional supplement/tool to guide and support their final decision. Numerical examples focusing on a multi-objective knapsack problem and an application to academic capital budgeting problem for project selection, are provided to verify the efficacy and added value of the methodology.

1. Introduction

Imperfect knowledge of the exact value of parameters, which comprises imprecision, ill-determination, and uncertainty (see [25]), is currently a major issue in mathematical programming. The obtained optimal solutions can exhibit remarkable instability and high vulnerability/volatility to changes in the values of the parameters of the problem, often rendering therefore a computed solution significantly suboptimal or not adequate for further implementation, (Bertsimas et al., [6]; Roy [24]; Ben-Tal et al., [3], etc). Therefore, the concept of robustness in mathematical programming has drawn the attention of the scientific community in this field and is usually set under the umbrella of "robust optimization" [3]. In a more or less informal way, by using the term "robustness" we actually mean that there exists some kind of imperfect knowledge in the model parameters and we examine ways and tools to stay "at the safe side" and safeguard decision making.

The degree to which a solution is stable to the underlying uncertainties within a model is usually defined as robustness. The concept of "robust optimization" in Operational Research was introduced by Soyster [27] but it was not until the last 20 years that it flourished and gathered the attention of the scientific community, mainly with the works by Mulvey et al. [21], Ben-Tal and Nemirovski [1,2] and Bertsimas and Sim [4,5]. Recently, Soyster and Murphy [28] also studied the concept of duality in robust optimization using linear programming. The concept of robustness in integer programming applications, such as product design has been studied by Wang and Curry [31], while Sawik [26] proposed a robustness approach for the supply chain problem. The reader is prompted to see Bertsimas et al. [6] and Gabrel et al. [12] for recent reviews on the subject.

Although robustness has been extensively studied in single objective mathematical programming problems, in the case of

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multi-objective optimization (MOO) several aspects remain to be explored. Kouvelis and Yu [15] in their seminal textbook devote a section to robustness and efficiency. A decade later, Deb and Gupta [7] introduced the concept of robustness in MOO using meta-heuristics. Some recent works also deal with robustness and MOO, for instance Zhen and Chang [32], where robustness is quantified and used as an additional objective function in a berth allocation problem. Roland et al. [22] provide a stability radius for the Pareto optimal solutions in multi-objective combinatorial problems. Roy [24] discusses the "multi-faceted" issue of robustness in the general context of operational research and not only in optimization. He initiates "robustness concern" and proposes a whole set of processes and actions when model parameters are imperfectly defined (p. 630). A concept of robustness in MOO was also introduced by Figueira et al. [10], especially in the case of interactive multi-objective optimization. Mavrotas et al. [20] have examined and analyzed the robustness of the most preferred efficient solution in MOO problems. Two recent works associated with robustness in multi-objective optimization are also worth mentioning: Ehrgott et al. [8] in a recent paper deal with the minmax approach of robustness, and Fliege and Werner [11] focused on robustness in a multi-objective context of portfolio selection.

In this paper we study the concept of robustness in multi-objective programming and especially in the generation (a posteriori) methods. These methods result in the generation of the whole set of efficient solutions (Pareto set) that includes all the Pareto optimal solutions (POS). The question that we attempt to answer in this paper is "How robust is the obtained Pareto set and the individual Pareto optimal solutions in the occurrence of changes or perturbations in model parameters?". We restrict our study to multi-objective combinatorial optimization (MOCO) problems, which, in their majority, concern multi-objective integer programming (MOIP) problems.

In our work we attempt to measure the robustness of POS, when uncertainty occurs by imprecision of the model's parameters. For this task we design an integrated methodology that can be applied in multi-objective discrete and combinatorial problems, using a combination of Monte Carlo simulation and optimization [30]. It must be noted that our approach does not constitute a sensitivity analysis over the results, where the instability of a single parameter is examined at a time. On the contrary, the use of Monte Carlo simulation, in SMAA method for instance [14], enables us to simultaneously alter the values of a number of parameters in a systematic way and extract holistic conclusions with respect to the robustness of the obtained solutions.

It is worth mentioning that Monte Carlo simulation has been also used for robustness analysis in multi-objective programming in the work of Mavrotas et al. [20]. However, in that work they studied the robustness of one specific Pareto optimal solution (most preferred solution) in relation to the preference parameters (weights) and not the robustness of the entire Pareto set in relation to the whole entity of the model's parameters. Since we are not practicing an exact method, but a simulation instead, we can refer to a pseudo-robustness analysis. Hereafter, we shall use the term robustness analysis, but we refer to a pseudo robustness analysis according to the terminology by Roy [23].

We denote as "reference Pareto set" the initial set of efficient solutions, the robustness of which we want to measure. In the proposed methodology we use Monte Carlo simulation in combination with the enhanced version of the ε-constraint generation method (AUGMECON2) that produces the exact Pareto set for MOIP problems [19]. Subsequently, we measure how many times a specific POS of the reference Pareto set is produced across the n Monte Carlo iterations. The higher the frequency, the more robust is the specific POS, since it exhibits a higher tendency to sustain its optimality. Consequently, besides the information regarding the performance of a POS to the criteria (objectives functions), we can provide the decision maker (DM) with an additional piece of post optimality information, namely the robustness measurements associated with perturbations in the model's parameters. A non-robust POS (i.e., it displays small appearance frequency in the Monte Carlo simulation–optimization process), signifies that it can be easily dominated by other solutions, when small perturbations in the model's parameters occur. Illustrative charts for problems with two and three objective functions are constructed, in order to depict the robustness of every POS in the reference Pareto front. Robustness indices for the POS as well as for the whole Pareto set are also calculated. In the end, we test the efficacy of the approach over two numerical examples and a case study regarding a capital budgeting problem for project selection with 108 binary decision variables.

The structure of the paper is as follows: In Section 2 we provide some basic concepts and definitions. In Section 3 we describe the methodology to measure robustness in MOCO problems. Section 4 illustrates two numerical examples in order to test the method, while Section 5 applies the method to an academic research proposal selection problem. Finally, in Section 6 we present the basic conclusions and discuss on some potential future perspectives of the work.

2. Concepts, definitions and notation

This section is devoted to some fundamental concepts on multi-objective combinatorial optimization, dominance and some of its other related concepts, robustness analysis or concerns, and some aspects on simulation.

2.1. MOCO problems

A multi-objective combinatorial optimization (MOCO) problem can be defined as follows:

Definition 1. (multi-objective combinatorial optimization problem). Let $I=\{1, 2,\ldots, I,\ldots,N\}$ denote a finite set of N objects or items, also called the ground set, and 2^I denote its power set (i.e., the set of all subsets of I), where $|2^I|=2^N$. Consider the subset $S\subseteq 2^I$ as the set of feasible solutions. Define the outcome/objective functions z_k: $I\to R$, such that the outcome vector of each solution $s\in S$ is as follows:

$$z(s)=(z_1(s),z_2(s),\ldots,z_k(s),\ldots,z_K(s)),\quad \text{where}\quad z_k(s)=\sum_{i\in S}c_{ik}$$

with c_{ik} being the value/outcome associate with each object $i\in S$, for kth objective function ($k=1, 2,\ldots,K$). The MOCO problem consists of finding a subset of feasible solutions, $F\in S$, when "maximizing" all the functions z_k, for $k=1, 2,\ldots,K$ (the sense of "maximizing" signifies that we search for a particular set of solutions called efficient solutions and defined in Section 2.2).

Any subset s of S is uniquely determined by its characteristic function Xs: $S\to\{0,1\}$ where $Xs(x)=1$ if $x\in S$ and $Xs(x)=1$ if $x\notin S$. With the help of this function, the problem of the above Definition 1 can be stated as a multi-objective optimization (MOO) problem:

$$\text{"maximize"}\{z_1(x),z_2(x),\ldots,z_k(x),\ldots,z_K(x)\}$$
$$\text{subject to}: x\in X\subseteq\{0,1\}^N$$

where $x=(x_1, \ldots, x_i, \ldots, x_N)$ is the vector of binary decision variables and X is the feasible region in the decision space. If the decision space is further described by the proper equalities/inequalities, the above MOO problem can be expressed as the following multiple