Omega 55 (2015) 53–60

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

A note on the facility location problem with stochastic demands ☆

Milena Bieniek

Statistic and Econometric Department, Economic Faculty, Maria Curie-Sklodowska University, Plac Marii Curie-Sklodowskiej 5, 20-031 Lublin, Poland

ARTICLE INFO

Article history:
Received 13 March 2013
Accepted 19 February 2015
Available online 25 February 2015

Keywords:
Location
Stochastic programming
Recourse

ABSTRACT

In this research note that the single source capacitated facility location problem with general stochastic identically distributed demands is studied. The demands considered are independent and identically distributed random variables with arbitrary distribution. The unified a priori solution for the locations of facilities and for the allocation of customers to the operating facilities is found. This solution minimizes the objective function which is the sum of the fixed costs and the value of one of two different recourse functions. For each case the recourse function is given in closed form and a deterministic equivalent formulation of the model is presented. Some numerical examples are also given.

1. Introduction

A general facility location problem (FLP) involves two given sets: a set of customers and a set of potential locations of facilities to serve customer demands (cf. [15]). The problem is to decide where to locate facilities and how to allocate customers to operating facilities. The decision is based on minimization of total service cost, but without assuming that customers' demands are known in advance. Therefore, the customers' demands must be considered as random variables and then FLP turns into facility location problem with stochastic demands (FLPSD).

In [2] the independent and identically distributed (i.i.d.) demands with Bernoulli distribution are considered. The authors called this problem facility location problem with Bernoulli demands (FLPBD). In this paper the FLPSD with arbitrary distribution of i.i.d. demands is studied.

Now we present the main assumptions of the model which are in general similar to the one presented in [2].

Assume that each facility has its own capacity i.e. the maximal total demand it can serve. On the other hand a facility will be opened in a given location only if some prescribed number of customers is assigned to it.

The FLPSD is modelled as two-stage stochastic program. In this program each customer must be assigned a priori to a single facility. The decision where to locate the facilities and how to allocate the customers to operating facilities must be made before knowing which customers in fact will be served. This is a strategic and tactical decision and it is made before knowing exactly what the demand will be it the future.

Since each customer's demand is uncertain, the overall demand of all customers assigned to a given facility may exceed its capacity and then we have to resort to outsourcing. Obviously, in this case some additional cost is incurred. Two outsourcing procedures are considered: customer outsourcing and facility outsourcing. Both of them apply outsourcing to satisfy the demand that exceeds the service capacity of open facilities.

In the first strategy, called customer outsourcing, outsourced demand is delivered directly to the customer at the cost dependent on the facility. Considering this strategy, it is assumed that requests of service are coming in a random order and the decision on the customers whose demand is outsourced is made on a first-come-first-served (FCFS) basis. In the second strategy, called facility outsourcing, the facility takes delivery of outsourced demand at a cost dependent on the facility. Then it delivers it to the customers at the same cost as if they are not outsourced. In both strategies the goal is to decide where to locate the facilities and how to allocate the customers in order to minimize the overall expected costs.

For more references of outsourcing we refer to [7] and [17]. In [17] zero-lead time strategy of outsourcing i.e. after demand realization and also non-zero lead time outsourcing, before demand realization, is considered. In [7] the manufacturing outsourcing and outsourcing of service process are studied.

In [2] each customer has unit or zero demand and the demands are independent random variables. It is suggested that it would be interesting to investigate other distributions of the demands.

We go further in our contributions. The obtained results are valid in the general case. All customers' demands are also i.i.d. random variables but with arbitrary probability distribution described by arbitrary distribution function. Moreover, the distribution function does not have to be discrete or continuous.

☆This manuscript was processed by Associate Editor Yagiura

http://dx.doi.org/10.1016/j.omega.2015.02.006
0305-0483/

It should be emphasized here that the problem with general distribution of demands is more complicated than in FLPBD. In [2] the number of customers with non-zero demands (called the demand customers) is equal to their total demand contrary to our case. In this note the number of the demand customers is not equal to their total demand in general. It produces many difficulties. As a result the received recourse functions are complicated but in a closed form. Although in some cases for specific distributions of demands the recourse functions' formulas simplify which is given in Section 5.

This is the theoretical research note and its aim is to find the explicit expressions for the recourse functions in general case.

Our contribution mainly lies in the following areas:

- We derive analytical solutions to the facility location problem with stochastic i.i.d. demands modeled by any distribution (not only discrete or continuous but possibly mixed).
- New random variable denoting the number of the demand customers has to be implemented. It has binomial distribution with known parameters.
- To derive the recourse functions the conditioning on the number of the demand customers is used instead of the conditioning on their total demand as it is done in [2].
- The formulas for the recourse functions can be simplified if the distribution of the sum of i.i.d. distributed demands is in closed form or if it is continuous distribution.

The problem in this note is the single source capacitated facility location problem with stochastic demands. The research literature on location models is vast. Facility location has a well-developed theoretical background and can be applied in many areas. Surveys on location studies are carried out among others in papers [14,15,32,33,38,39,41] and [29]. In the latter paper continuous sequential competitive location problems on networks are studied. Recent and relevant literature on location and related models and the methods used to solve them are as follows.

In [40] the author proposes solution to capacitated facility location problem giving a heuristic algorithm. The paper [36] presents a strategic solution of FLP incorporating external and internal criteria in the decision making process. Also the strategic and tactical aspects with two stage stochastic program are considered in [6] and [44]. In the latter paper the optimal location for oil spill response facilities is found. Location and allocation model of specialized health is developed and solved in [24,34] or [43].

Stochastic features are introduced into facility location model in [18]. Some authors propose also queueing location models [19] and models connected with game theory [25]. Additionally, the paper [31] is addressed to the stochastic uncapacitated location problem with random demands, selling price and some other factors. Here a scenario approach is adopted to this problem and a heuristic dual based procedure is used to solve it. Also [30] is devoted to FLP with stochastic demand.

An optimal design of supply chain including locating new facilities is treated in [20], where the demand is time varying and the uncertainty is captured in terms of likely scenarios. Also the location of new facilities for chain expansion is the subject of [37].

The location and routing problem with pickup and delivery is formulated in [26]. The authors also propose a heuristic approach to solve large-size problems.

The combination of facility location and network design is presented in [12,13] or [35]. In this case the location model has various applications in regional planning, telecommunication, electrical distribution system, emergency system and other areas.

Furthermore the location models are also employed to reverse logistics which is described in [11] and [42].

Dynamic approach to FLP is studied for instance in [3,4,21] and [28]. In [21] the authors adopted facility location to location of mobile facilities in railway construction. The facility location in railway network is also considered in [10].

Recently, the capacitated k-facility location problem is studied in [1], while the heuristic method for solving single source capacitated FLP is given in [23]. Moreover in [8] a continuous location problem on a line is considered together with facility reliabilities.

Finally, some authors discussed robustness in facility location problems (see [5,16,22]). In [5] the robust optimization is applied to FLP under uncertain demand over multiple periods. Using robustness in [22] the authors find the solution of FLP considering normal or ambiguous distributed demand. In [5] and [22] different problems than in our note are studied. First of all they differ with the assumptions of the models and they use a robust approach. The problem in [22] is simplified to the nonlinear program but here the linear one is presented. Moreover the normal or ambiguous distribution is studied but with very restrictive assumptions on the parameters. Similarly in [5] the distribution with the parameters on the some specific set is considered.

In general, facility location problem studied in this paper belongs to the class of two-stage stochastic programs with recourse (see [9,33]). More precisely, the problem under consideration here is to define a recourse action which, given a priori solution, indicates the best possible realization of the random vector. Then one has to look for the a priori solution that minimizes the expected cost of recourse action.

The rest of this paper is organized as follows. In the next section the definition of the problem is restated and the used notation is defined. In Section 3 we derive the recourse functions for each of the two strategies of outsourcing and we present them in closed forms. Next in Section 4 the deterministic equivalent formulations for the FLPSD are given. In Section 5 the explicit formulas for the recourse functions in special cases of the distribution of the demands are presented. Some numerical examples are given in Section 6. Finally, Section 7 concludes this research note.

2. The definition of the problem

In this section we restate the model of [2] completed with some our new variables necessary in the consecutive considerations.

Let I, with $u=|I|$, be the set of indices for the potential locations of facilities and let J, with $n=|J|$, be the set of indices of customers. The indices do not have to be the consecutive natural numbers. We use the following notations:

- ξ_j is the size of demand of the customer $j \in J$; we assume that ξ_j, $j \in J$, are i.i.d. random variables with common distribution function F and finite mean $E(\xi_j)=\mu$ for all $j \in J$;
- f_i, $i \in I$, is the fixed set-up cost of opening facility i;
- ℓ_i, $i \in I$, is the minimal number of customers that must be assigned to facility i to be opened;
- D_i, $i \in I$, is the maximal total demand of customers that can be served by facility i when it is opened. The quantity D_i is called the capacity of plant i;
- c_{ij}, $i \in I$, $j \in J$, is the cost of serving one unit of demand of customer j from facility i;
- g_i, $i \in I$, is the cost of supplying of a unit of outsourced demand in customer outsourcing strategy;
- $\hat{g}_i$, $i \in I$, is the cost of supplying of a unit of outsourced demand in facility outsourcing strategy.

We stress that no further assumptions are made on the random variables ξ_j, $j \in J$, other than independence and stationarity of distribution.