Omega 55 (2015) 81–90

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

The consensus models with interval preference opinions and their economic interpretation ☆

Zaiwu Gong [a,b,*], Xiaoxia Xu [b], Huanhuan Zhang [b], U. Aytun Ozturk [b], Enrique Herrera-Viedma [c,d], Chao Xu [b]

[a] Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters, Nanjing University of Information Science and Technology, Nanjing 210044, China
[b] School of Economics and Management, Nanjing University of Information Science and Technology, Nanjing 210044, China
[c] Department of Computer Science and Artificial Intelligence, University of Granada, Granada 18071, Spain
[d] Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 14 October 2014
Accepted 9 March 2015
Available online 16 March 2015

Keywords:
Group decision-making
Multi-objective optimization
Consensus opinion
Interval preference
Minimum cost
Maximum return

ABSTRACT

This paper aims to explore the case when an individual opinion is interval preference in consensus decision making. And for this purpose, we construct two multi-objective optimization models: one based on the minimum cost from the perspective of the moderator, the other the maximum return from the perspective of the individuals. On the basis of multi-objective programming theories, these multi-objective programming models are then transformed into two single-objective linear programming models, i.e., the primal model and the dual model. The primal model focuses on how to obtain a consensus with the minimum cost, while the dual model is concerned with how to get the maximum return. With the help of dual linear programming theories, we have revealed the following economic significance of the primal-dual consensus models: the primal-dual consensus models can not only help us probe into the relations between the minimum cost paid by the moderator and the maximum return expected by individuals who changed their opinions before, but also help us explore the relations between the unit cost that the moderator pays each individual, unit return that each individual receives, each individual opinion and the consensus opinion. This paper with the aid of theoretical analysis and an illustrative example indicates that once the consensus is obtained, the optimal unit return and optimal consensus opinion value are also solved. This paper also points out that the amount of the total return acquired by all the individuals who have abandoned their original opinions before is equivalent to that of the total cost paid by the moderator to reach the consensus. This paper also argues that there exists compact correlations between the individual's unit return, the consensus opinion, the individual's interval opinion, and the moderator's unit cost.

1. Introduction

Group decision making (GDM) mainly focuses on unstructured decision making problems that require experts' subjective judgment [1–4]. Generally, experts consist of lots of individual decision makers (DMs) who represent different interests, values and preferences. And therefore, how to arrive at a consensus has become a hot topic in GDM research. The definition of consensus has been varied widely. The American Heritage Dictionary defines consensus as “an opinion or position reached by a group as a whole”. Bezdek et al. [5] and Spillman et al. [6] interpreted consensus as “a full and unanimous agreement”. In fact, these two definitions are concerned with the final outcome, while Ref. [7–9] stress the evolutionary process of reaching consensus. Besides, Ness et al. [10] take consensus to represent the case where most DMs “agree on a clear option”, the few DMs who oppose this option provide rational and essential suggestions, and eventually, all the DMs “agree to support the decision”. In addition, Steve [11] divided consensus into two categories: accidental consensus (i.e., the theory chosen by all individual DMs based on their own independent judgment) and essential consensus (i.e., the viewpoint determined by collective negotiations and discussions). Obviously, how to reach the consensus in the latter category actually involves a multistage process: each individual

☆This manuscript was processed by Associate Editor Doumpos.

* Corresponding author at: Collaborative Innovation Center on Forecast and Evaluation of Meteorological Disasters, Nanjing University of Information Science and Technology, Nanjing 210044, China.
E-mail address: zwgong26@163.com (Z. Gong).

http://dx.doi.org/10.1016/j.omega.2015.03.003
0305-0483/

changes their opinions gradually and all the views tend to be unanimous after many rounds of discussion [12,13]. More importantly, the above evolutionary process usually needs a worthy, effective and efficient moderator (i.e., super-individual defined by [14]) who dominates the whole process of consensus reaching and has strong team leadership and communication skills to convince individuals to change their opinions into an acceptable option [12,14]. Therefore, this research is based on the following hypothesis: a moderator [15,16] introduced in GDM is able to persuade DMs to change their opinions towards a consensus opinion by paying costs (i.e., consuming resources such as time or money), and DMs' opinions will gradually approach the consensus.

Actually, consensus decision making involves a multi-objective problem-solving process, in which each individual is the minimum decision making unit, and his/her opinion can be regarded as a sub-objective. In the multi-objective optimization theory, a non-inferior (but not globally optimized) solution that meets most goals is a satisfying choice and is good enough for many real-life situations. Generally, multi-objective problems are often transformed into single objective problems with some decision rules [17]. Such aggregation operators in GDM as the weighted averaging (WA) operator [18], the ordered weighted average (OWA) operator [19,20], the power average (PA) operator [21], and the probabilistic weighted average operator [22], representing decision rules in mathematics, are determined by DMs or the moderator under specific GDM background. For example, Ben-Arieh and Chen [23] presented a method which aggregates experts' judgements into a collective opinion with the fuzzy linguistic OWA operators and calculates the consensus level. Zhang [24] adopted several new hesitant fuzzy aggregation operators to solve multiple attribute GDM problems in hesitant fuzzy environments. His research incorporates both the decision arguments and the relationships between them. Furthermore, Parreiras et al. [25] introduced a flexible consensus scheme for multi-criteria GDM under linguistic assessments based on fuzzy aggregation models. In fact, the optimal solution to a single objective problem is actually a Pareto optimal (non-inferior) solution to the corresponding multi-objective problem [26], meaning that an ideal consensus reached in GDM is merely an optimal solution in the single-objective decision making sense, or merely a Pareto optimal solution in the multi-objective decision making sense.

In 2007, Ben-Arieh and Easton [15] introduced the concept of minimum cost consensus, constructed a multi-criteria consensus model under linear cost opinion elasticity, and presented linear-time algorithms to find the minimum cost consensus. They then generalized their work to derive new algorithms for reaching it [16]. The minimum cost consensus model by Ben-Arieh and Easton [15,16] is to aggregate the deviations between an individual's opinion and consensus opinion using a weighted arithmetical mean operator and to construct an optimization programming model based on it. In fact, the consensus model, proposed by Ben-Arieh et al., stems from Gonzalez-Pachon and Romeroc's distance-based goal programming (GP) models [4,27,28]. The widely used standard GP model, brought forward by Charnes and Cooper [29], aims to minimize the deviations attached to the goal and the aspiration levels determined by DMs. As a result, Gonzalez-Pachon and Romeroc's findings have laid a theoretical foundation for the research of Ben-Arieh et al. on the construction of the minimum cost consensus models.

The earlier consensus research based on total cost only takes into account the moderator's point of view, while in fact, individuals have to continuously modify their opinions to obtain a compromise consensus [28], and therefore it is necessary to take their interests into consideration as they deserve to be compensated. A case in point is the realization of IPCC (Intergovernmental Panel on Climate Change) consensus, which is obtained through multiple rounds of negotiations and constant compromises among political groups as well as experts in various fields. In the process, the consensus will be unilateral, or may be swayed by sectional groups if only the viewpoints of some center groups (i.e., moderator) are considered. And hence, to obtain a more scientific IPCC consensus, we should consider the voice of the developing countries or relatively weak groups (i.e., individual DM) who have abandoned their own interests to arrive at the generally accepted result. As an exchange, they will expect some compensation (e.g. lower taxes or more carbon quotas) and the more the better. Therefore, this paper constructs two linear optimization models based on the minimum cost and maximum return to explore from both the moderator's and the individual DMs' perspectives the cost consensus problems in GDM. With the help of the primal-dual linear programming theory, we analyze the relationships between all the variables and present the economic interpretation of the models proposed. This paper is an extension of the minimum cost consensus model presented by Ben-Arieh and Easton [15,16].

The rest of this paper is arranged as follows. Section 2 introduces the distance measure on consensus decision making and then proposes two multi-objective programming models based on minimum cost and maximum return. Section 3 establishes the minimum cost and maximum return models for reaching consensus, which are based on primal-dual theory; and also explores the economic significance of these two models. Section 4 investigates the properties of these two primal-dual linear programming models and further discusses the theoretical meaning and economic significance of the primal-dual models. To further explain the proposed models, we provide an illustrative example in Section 5. Lastly, the conclusion and directions for future research are provided in Section 6.

2. Multi-objective programming models based on minimum cost and maximum return

Suppose that there are m decision makers (DMs) $D=\{d_1,\ldots,d_m\}$ taking part in a GDM. Let o_i, $o_i \in R$, represent the opinion of DM d_i, $i=1,\ldots,m$, in the GDM. Each opinion of an individual DM represents an individual interest, so we define it to be an individual opinion. In consensus decision making, the ideal state is where there exists an ideal opinion o' such that $o_1=\ldots=o_m=o'$. That is, when all opinions are equal to the same ideal opinion o', the group has arrived at unanimity. Such an ideal opinion in fact represents the collective interest, so we define it as a consensus opinion ([15,16] define it when all DMs have the same current group opinion). In reality, it is difficult to obtain such a perfectly identical opinion, even if all individuals have similar values, backgrounds, abilities, knowledge structures, experiences, and so on. On the one hand, to arrive at a consensus, the moderator in GDM believes that he/she can persuade each individual to change his/her opinion to an ideal value by paying a cost (consuming resources such as time or money). On the other hand, all individuals expect to obtain return for changing their opinion to the ideal one (the consensus opinion). In other words, during the process of reaching consensus, the moderator expects to pay a cost to obtain consensus and each individual hopes to receive compensation because he/she has sacrificed his/her interests for the collective interest. In consequence, we construct a deviation function f to measure the changes between an individual opinion and the consensus opinion. Meanwhile, a unit cost w is paid by the moderator to persuade each individual to change his/her opinion. The moderator represents the collective interest, so it is natural that he/she hopes to pay as little as possible, while the individual cares for his/her own interest, and so obviously expects to gain return as large as possible.