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Robust multi-criteria sorting with the outranking preference model and characteristic profiles

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ABSTRACT

We present a new multiple criteria sorting approach that uses characteristic profiles for defining the classes and outranking relation as the preference model, similarly to the Electre Tri-C method. We reformulate the conditions for the worst and best class assignments of Electre Tri-C to increase comprehensibility of the method and interpretability of the results it delivers. Then, we present a disaggregation procedure for inferring the set of outranking models compatible with the given preference information, and use the set in deriving, for each decision alternative, the necessary and possible assignments. Furthermore, we introduce simplified assignment procedures and prove that they maintain a no class jumps-property in the possible assignments. Application of the proposed approach is demonstrated by classifying 40 land zones in 4 classes representing different risk levels.

1. Introduction

In the multiple criteria sorting problem (also called ordinal classification), decision alternatives are assigned to one or more homogeneous classes based on their evaluations on multiple attributes. The classes considered here are ordered and pre-defined, which means that they, unlike clusters [1], do not result from the analysis. For example, the submitted papers need to be assigned to categories reject, weak reject, weak accept, or accept, and employees may be acknowledged for their performance with a low, medium or high bonus. Indeed, sorting is an important decision problem in fields such as finance [2,3] and environmental risk assessment and management [4–6].

In this paper we consider multi-criteria sorting problems applying the non-compensatory outranking preference model (for some recent advances and applications of outranking-based approaches see [7–11]). The most well-known among such methods is Electre Tri-B [12] that employs boundary profiles for modeling the frontiers between two consecutive classes. However, in some decision situations the frontiers between the classes have no objective existence because the separation between the consecutive classes can be conceived in several ways. In Electre Tri-C [13] the alternatives are not compared against the class boundaries, but rather with characteristic profiles that are formed from the representative attribute values for each class. For each decision alternative, Electre Tri-C results in an assignment in form of an interval of classes. The aim of the current paper is to make Electre Tri-C more usable in real-life analyses by introducing the following four advances.

First of all, we reformulate the two Electre Tri-C assignment procedures whose outcomes delimit a resulting interval of classes for each alternative. Depending on the results of a comparison of an alternative with the characteristic profiles, the order of classes indicated by the descending and ascending assignment rules may vary. That is, with some outranking models the ascending rule may indicate an assignment to a better class than the descending rule, whereas with other models the order can be reversed. While respecting the assumptions and providing the same results as Electre Tri-C, the reformulated assignment procedures indicate the lower and upper classes unambiguously. Discovering the precise conditions for the extreme class assignments is important for two reasons. First, it increases the transparency of the method. Second, the reformulated procedures support the explanation of the results in natural language, thus increasing the comprehensibility of the sorting recommendation.

The second aim of the paper is to introduce a disaggregation procedure for inferring the parameters for the preference model used in Electre Tri-C. As in other sorting methods that apply the outranking preference model, it is not realistic to assume that the Decision Maker (DM) can provide exact values for all the parameters. Some of them can be defined by the DM fairly easily

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(comparison thresholds, class profiles) [12], but others (criteria weights and the majority threshold) are harder to elicit [14–16]. Although Electre Tri-C has been found appropriate for the context of real-world problems in project management [17], environmental modeling and assessment [18,19], tourism [20], and medical diagnosis [21], most of these studies report problems settings with precise values for the inter-criteria model parameters (for discussion on eliciting precise values for the weights and the majority threshold in Electre Tri-C, see, respectively, [21] and [17]).

We generalize the disaggregation procedure to account for imprecise assignment examples. In this way, we provide the first disaggregation approach for outranking-based sorting methods that admits specification of interval assignments. This increases the flexibility of the preference elicitation process by allowing preference statements reflecting hesitation with respect to the desired assignment (e.g., "*a* should be assigned to class medium or good") as well as these concerning classes excluded from the set of desired assignments (e.g., "*b* should not be assigned to class bad"). The main challenge in designing a disaggregation procedure for Electre Tri-C is that its assignment rules indicate an interval of classes. Due to this imprecision the conditions for assigning an alternative a to a certain interval of classes $[C_{L^{DM}}, C_{R^{DM}}]$, $L^{DM} \le R^{DM}$, cannot be derived directly from the respective sorting rules. Instead, one must guarantee that a is assigned neither to a class worse than $C_{L^{DM}}$ nor better than $C_{R^{DM}}$.

Our procedures infer a set of Electre Tri-C models from assignment examples. To avoid an arbitrary definition or selection of a single model for the final classification, the third aim of the paper involves the adaptation of the Robust Ordinal Regression (ROR) methodology [16,22–25] to outranking-based multiple criteria sorting. By exploiting the whole set of compatible Electre Tri-C models, we compute the necessary and possible assignments for each alternative, that is, assignments that hold for all or at least one compatible model, respectively. The necessary and possible assignments are computed through Mixed-Integer Linear Programming (MILP).

Using the proposed approach, the DM can initially introduce only a few representative reference alternatives that she considers appropriate to be assigned to some classes. Then, we allow the progressive incorporation of preference information in the context of an outranking model assumed by Electre Tri-C, which makes it easier to associate each assignment example individually with changes in the necessary and possible assignments. Analysis of these results may stimulate the DM to interactively specify possibly imprecise assignment examples. Such interaction can be particularly useful for constructive preference learning [22]. Let us note that in the recent Electre Tri-nC method [26] each class may be also defined with several characteristic profiles. Contrary to our proposal, Electre Tri-nC requires the profiles to be provided at once, and employs them within a standard aggregation procedure for computing the alternatives' final assignments.

The fourth aim of this paper is to introduce revised, simpler versions of Electre Tri-C assignment procedures, called Electre Tri-rC. The results obtained with the revised procedures differ from those of Electre Tri-C only in very specific problem instances. The new procedures have significant implications for the ROR approach because the space of compatible outranking models is now convex. Consequently, we are able to prove a no class jumps-property for the possible assignments computed with the revised procedures, leading to easier interpretation of its results. Let us emphasize that this desirable property does not hold for the possible assignments obtained with a set of Electre Tri-B models (see [27]).

We continue by introducing the outranking preference model in Section 2 and rest of the Electre Tri-C method in Section 3. Section 4 presents the disaggregation approach for Electre Tri-C, procedures for computing the necessary and possible assignments, and the reformulated assignment procedures. Section 5 introduces Electre Tri-rC, and adapts the previous formulations to the revised assignment procedures. Section 6 describes a decision aiding process for the proposed approach. Section 7 demonstrates the use of the approach by analyzing an example. Section 8 concludes.

2. The outranking preference model

We use the following notation:

- $A = \{a_1, a_2, \ldots, a_i, \ldots\}$ – a set of decision alternatives.
- $F = \{g_1, \ldots, g_j, \ldots, g_n\}$ – a consistent family of n criteria, $g_j : A \to \mathbb{R}$; we assume, without loss of generality, that all criteria are maximized, i.e. the attractiveness increases with the criterion performance increase.
- $C_1, \ldots, C_h, \ldots, C_t$ with $t \ge 2$ – a set of pre-defined completely ordered (from the worst to the best) classes so that C_{h+1} is preferred to C_h, $h = 1, \ldots, t-1$; each class is defined with a characteristic profile[1] b_h.
- $A^R = \{a_1^*, a_2^*, \ldots\}$ – a finite set of reference alternatives on which the DM accepts to express preferences. We assume that $A^R \subseteq A$.

The binary outranking relation corresponding to the statement "at least as good as" is denoted by xSy, and its negation by xS^cy. The outranking preference model applies pseudo-criteria [28] that models per-criterion attractiveness with indifference ($q_j(x)$) and preference ($p_j(x)$) thresholds defined either as affine functions or constant values so that if

$$\begin{array}{ll} |g_j(x) - g_j(y)| \le q_j(x), & x \text{ is indifferent to } y, \text{ denoted } xI_jy, \\ g_j(x) - g_j(y) \ge p_j(x), & x \text{ is strictly preferred to } y, \text{ denoted } xP_jy, \\ q_j < g_j(x) - g_j(y) < p_j(x), & x \text{ is weakly preferred to } y, \text{ denoted } xQ_jy. \end{array} \tag{1}$$

A set of importance coefficients (weights) $w_j \ge 0$, $j = 1, \ldots, n$, is associated with the set of criteria. Without loss of generality, we assume that $\sum_{j=1}^{n} w_j = 1$. Computing the outranking relation for a pair (x, y) involves the computation of a comprehensive concordance index $c(x, y)$, which represents the strength of the coalition of criteria being in favor of xSy:

$$c(x, y) = \sum_{j=1}^{n} c_j(x, y) = \sum_{j=1}^{n} w_j \cdot \varphi_j(x, y), \tag{2}$$

where

$$\varphi_j(x, y) = \begin{cases} 0 & \text{if } g_j(y) - g_j(x) \ge p_j(x), \\ 1 & \text{if } g_j(y) - g_j(x) \le q_j(x), \\ [p_j(x) - (g_j(y) - g_j(x))]/[p_j(x) - q_j(x)] & \text{if } q_j(x) < g_j(y) - g_j(x) < p_j(x). \end{cases} \tag{3}$$

Veto thresholds $v_j(x)$ such that $v_j(x) \ge p_j(x)$, can be used to model the effect that an alternative cannot be at least as good as another if it has too low performance in even one of the attributes, i.e., the criterion "vetoes" against the outranking. When these are used, the outranking computation needs to take into account additionally the discordance indices $d_j(x, y)$:

$$d_j(x, y) = \begin{cases} 1 & \text{if } g_j(y) - g_j(x) \ge v_j(x), \\ 0 & \text{if } g_j(y) - g_j(x) < v_j(x). \end{cases} \tag{4}$$

[1] In [13] the b_h, $h = 1, \ldots, t-1$, are called characteristic reference alternatives, but to avoid confusing them with the reference alternatives A^R for which the DM provides assignment examples, we call them characteristic profiles.