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The two-dimensional vector packing problem with piecewise linear cost function ☆

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ABSTRACT

The two-dimensional vector packing problem with piecewise linear cost function (2DVPP-PLC) is a practical problem faced by a manufacturer of children's apparel that ships products using courier service. The manufacturer must ship a number of items using standard-sized cartons, where the cost of a carton quoted by the courier is determined by a piecewise linear function of its weight. The cost function is not necessarily convex or concave. The objective is to pack all given items into a set of cartons such that the total delivery cost is minimized while observing both the weight limit and volume capacity constraints. This problem is commonly faced by many manufacturers that ship products using courier service. We formulate the problem as an integer programming model. Since the 2DVPP-PLC generalizes the classical bin packing problem, it is more complex and challenging. Solving it directly using CPLEX is successful only for small instances. We propose a simple heuristic that is extremely fast and produces high-quality solutions for instances of practical size. We develop an iterative local search algorithm to improve the solution quality further. We generate two categories of test data that can serve as benchmark for future research.

1. Introduction

We consider a new extension to the bin packing problem, the two-dimensional vector packing problem with piecewise linear cost function (2DVPP-PLC). In the 2DVPP-PLC, a set of items are to be packed into identical bins so that the total cost of utilized bins is minimized. Each item has two attributes: the weight and the volume. A set of items can be packed into a bin if their total weight and volume do not exceed the capacities of the bin. The cost of a bin is a piecewise linear function of the total weight of packed items.

This problem is motivated by a project awarded by a manufacturer of children's apparel with several production bases and hundreds of retail stores located across the globe. The manufacturer distributes its products from its production bases to retail stores through an express courier company such as Federal Express or DHL under a long-term contract. Articles of children's apparel (such as shorts, jackets and rompers) of the same style and size are bundled to form items; for example, one item may be a bundle of two dozen rompers. In each delivery, a batch of items produced by a production base are packed into cartons and delivered to a specific store by a courier company. To simplify the operations of packing, storage and transportation, the manufacturer employs only one type of carton with fixed dimensions in each delivery.

The delivery cost for a carton is calculated based on the weight of the carton, the source and the destination. The destinations (stores) are grouped into zones. For a given production base and the destination zone, the delivery cost for a carton is a function of the total weight of the items packed into that carton (we assume that the net weight of the carton is negligible). We obtained from the manufacturer the pricing function offered by the courier company to deliver one carton from a certain production base to each of the seven zones, as shown in Fig. 1(a), where the x-axis represents the total weight of the items in pounds (lb) in a carton and the y-axis corresponds to the delivery cost of that carton in US dollars. We approximate the pricing function for each zone with a piecewise linear cost function. For example, the pricing function for Zone 3 is approximated by the following function (Fig. 1(b)):

$$f(x) = \begin{cases} 0 & \text{if } x = 0; \\ 5 & \text{if } x \in (0, 10]; \\ 0.2x + 3 & \text{if } x \in (10, 70]; \\ 0.5x - 18 & \text{if } x \in (70, 150]. \end{cases} \quad (1)$$

We have conferred with the courier company on the rationale behind this pricing scheme, which was devised based on practical

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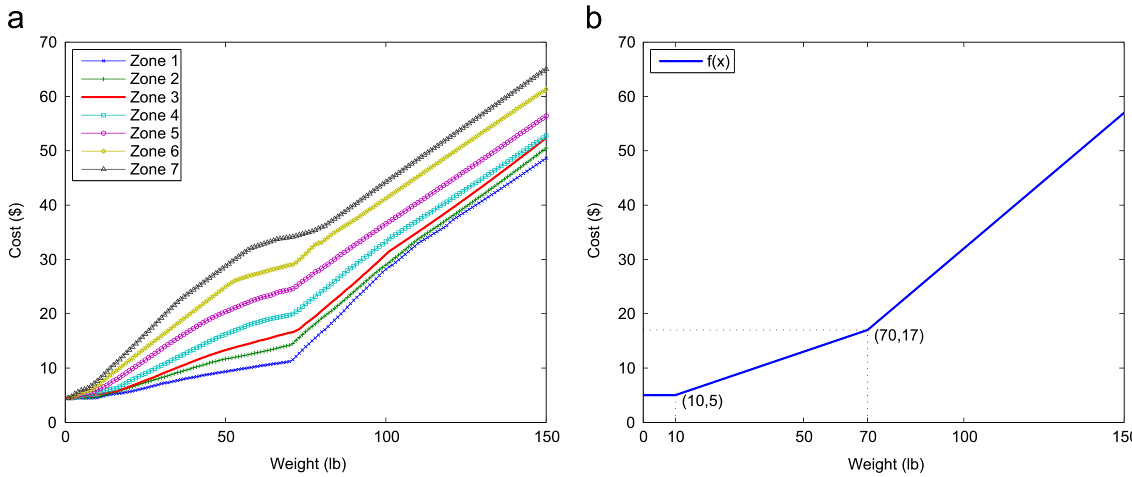

Fig. 1. (a) Pricing schemes for seven zones. (b) Piecewise linear function for Zone 3.

considerations. Delivering each carton incurs a fixed transaction cost, so the courier company charges US$5 for cartons with weight not more than 10 lb. When the carton weight exceeds 10 lb, the delivery cost increases with weight at a fixed rate of 20 cents per lb. However, this rate increases to 50 cents per lb when the carton weight is greater than 70 lb. One of the reasons is that light cartons can be handled manually by a single worker with ease, while overweight cartons require special handling equipment or more than one worker, which increases the operation cost considerably. Moreover, the carton weight is capped by a specific limit (150 lb in our case).

Since textile packages are flexible and non-fragile, the manufacturer only needs to consider the weight and the volume of each item during the packing process, i.e., we can safely assume that a set of items can fit into the carton as long as the total volume and weight of the items do not exceed the volume capacity and the weight limit of the carton, respectively. Therefore, minimizing shipping cost of a delivery can be modeled as the two-dimensional vector packing problem with piecewise linear cost function (2DVPP-PLC).

Both the two-dimensional vector packing problems (and many related variants of bin packing problems) and the piecewise linear cost function (in the context of optimization problems) have received considerable attention (Section 2). However, there is no prior literature dedicated to the 2DVPP-PLC, to the best of our knowledge, despite its value to practitioners. We, therefore, initiate the investigation of this problem by formally introducing the problem and presenting an integer programming (IP) formulation (Section 3). The 2DVPP-PLC as a generalization of bin packing problem is clearly NP hard. Solving the IP formulation using standard IP solver, such as CPLEX, is viable only for small problem instances. We resort to heuristics to produce practical solutions for large instances for our client. We adapt the classical first-fit and best-fit heuristics to generate initial solutions (Section 4). Although first-fit and best-fit produce good initial solutions, both are incomplete in the sense that all optimal solutions may be missed for some problem instances. We extend a shortest-path based heuristic from Haouari and Serairi [14] to the 2DVPP-PLC, which is complete (Section 5). Based on this shortest-path heuristic, we develop an iterative local search algorithm to improve the solution quality (Section 6). We generate two categories of test instances: **opt**, where an optimal solution is known; and **rand**, where no optimal solution is known. Computational experiments on **opt** instances show that our proposed algorithm is capable of finding high-quality solutions in a reasonable computation time. The category **rand** includes a comprehensive set of instances of various scales and can serve as benchmark for future research.

2. Literature review

Packing and cutting are important processes in production and logistics. There is a large number of publications in the area of packing and cutting [8,25,2] over the decades. Wscher et al. [30] provided a typology to organize and categorize existing literature.

The 2DVPP-PLC is one of many generalizations of the classical one-dimensional bin packing problem (BPP) [9,12]. The rich body of literature on many variants of the BPP includes other generalizations such as the two-dimensional (rectangle) bin packing problem (2D-BPP) [22,23], which extends both bins and items to be rectangles and requires that the items must be orthogonally packed into the bins without overlapping; the three-dimensional (rectangle) bin packing problem (3D-BPP) [21,20,32,31,5], which attempts to orthogonally pack boxes into containers without overlapping; the bin packing problem with conflicts (BPP-conflicts) [13,24], which specifies that some pairs of items cannot be loaded into the same bin; the two-dimensional vector packing problem (2DVPP) [26,3], which adds a second attribute (in addition to volume) and requires that the total for both attributes in a single bin must be within the specified limits; and the variable-sized bin packing problem (VSBPP) [17,14,15,28], which considers different sizes of bins each with a different specified cost; the bin packing problem with general cost structure (BPP-GC) [1,10], which introduces a general cost structure when calculating the cost of a bin.

The 2DVPP-PLC generalizes the 2DVPP. While the 2DVPP aims to minimize the number of bins used, the 2DVPP-PLC extends and targets to minimize the total cost which depends on the given piecewise linear cost function. In the 2DVPP-PLC, minimizing cost does not imply that the number of bins used is minimized. In fact, splitting an overly filled bin into two bins may reduce the total shipping cost (we provide an illustrative example of the difference between the 2DVPP and the 2DVPP-PLC in Appendix A in the online supplements). In the existing literature, the 2DVPP has been solved mainly using two classes of algorithms: exact algorithms and heuristics. Woeginger [29] proved that there is no polynomial-time approximation algorithm with a performance guarantee of less than or equal to $1+\epsilon$ (where ϵ tends to zero) for the 2DVPP. Subsequently, Kellerer and Kotov [18] designed an $O(n \log n)$ approximation algorithm that has an absolute worst-case