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A comparison of fixed and dynamic pricing policies in revenue management

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ABSTRACT

We consider the problem of selling a fixed capacity or inventory of items over a finite selling period. Earlier research has shown that using a properly set fixed price during the selling period is asymptotically optimal as the demand potential and capacity grow large and that dynamic pricing has only a secondary effect on revenues. However, additional revenue improvements through dynamic pricing can be important in practice and need to be further explored. We suggest two simple dynamic heuristics that continuously update prices based on remaining inventory and time in the selling period. The first heuristic is based on approximating the optimal expected revenue function and the second heuristic is based on the solution of the deterministic version of the problem. We show through a numerical study that the revenue impact of using these dynamic pricing heuristics rather than fixed pricing may be substantial. In particular, the first heuristic has a consistent and remarkable performance leading to at most 0.2% gap compared to optimal dynamic pricing. We also show that the benefits of these dynamic pricing heuristics persist under a periodic setting. This is especially true for the first heuristic for which the performance is monotone in the frequency of price changes. We conclude that dynamic pricing should be considered as a more favorable option in practice.

1. Introduction

Pricing is one of the most important decisions that impact a firm's profitability. The effect of pricing is more profound for companies in transportation services sector where it is difficult to change capacities in the short term and variable costs are small. Recognizing this, airlines, rental car companies and other firms in transportation and service industries have begun to implement techniques to improve their pricing and allocation decisions since mid 1980s. Following the success of these practices, now broadly called revenue management, pricing decisions are becoming more tactical and dynamic pricing is increasingly being adopted in retail and other industries.

In a seminal work, Gallego and van Ryzin [1] (GvR hereafter) study the problem of dynamically pricing a fixed stock of items over a finite horizon under uncertain demand. An important result in GvR is that keeping the price constant (at a level determined by the deterministic solution of the problem) throughout the horizon has a bounded worst-case performance and is asymptotically optimal as the expected sales goes to infinity. GvR also show numerically that when the demand function is exponential, fixed-price policies have good performance even when the expected sales is small. The authors conclude that "…offering multiple prices can at best capture only second-order increases in revenue due to the statistical variability in demand". Since 1994, a large and important body of literature in operations research has evolved to offer solutions and study different variants of the problem studied in GvR. (Recent examples include research that study the impact of product substitution [2], consumer inertia [3] and competition and price uncertainty [4] on dynamic pricing. See [5–7] for extended reviews of earlier literature.) Although GvR caution that these second-order increases in revenue may be significant in practice, revenue management literature has remained relatively silent on quantifying the benefits of dynamic pricing over fixed-price policies. This is primarily due to practical convenience: computing optimal dynamic prices is difficult (if not impossible) and changing prices frequently may be undesirable or costly.

Our primary aim in this paper is to reemphasize the power of dynamic pricing under resupply restrictions. We suggest two computationally simple dynamic pricing heuristics and show that the performance of these heuristics can be significantly better than that of fixed-price policies. In particular, we first propose the *revenue approximation* heuristic which is based on approximating the expected revenue of the optimal policy in order to calculate the price to be applied for a given remaining inventory and remaining time in the selling season. The approximation is a combination of a lower bound based on the homogeneity of the optimal expected revenue and an upper bound based on the deterministic version of the problem. The second heuristic we

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suggest is the *dynamic run-out rate* heuristic which adaptively uses the solution of the deterministic version of the problem. We carry out an extensive numerical study which shows that the revenue gap between fixed-price and optimal dynamic pricing policies may be substantial and this gap worsens when the season length (or demand potential) increases. We show that the two heuristics that we propose close a significant portion of this gap and lead to near-optimal expected revenues. We also show that most of the benefits of dynamic pricing heuristics are sustained by changing the prices periodically rather than continuously. For the first heuristic, the performance is monotone in the number of periods used. Our analysis and results are confined to the benefits of dynamic pricing under "normal" statistical fluctuations in demand. The benefits of dynamic pricing will be more pronounced when the demand is non-homogeneous or when the demand function or distribution is not known in advance.

Among the relevant works in the literature, Gallego and van Ryzin [8] extend their model to the multiple products case and demonstrate that two heuristics that are similarly based on the solution of the deterministic version of the problem are asymptotically optimal. Cooper [9] proves asymptotical convergence results that are stronger than those in GvR and [8]. Cooper [9] also presents an example where updating prices (more precisely, the allocations in Cooper's model) by resolving the deterministic problem throughout the horizon, a widely applied approach in practice, may perform worser than applying the static policy. Secomandi [10] establishes the conditions under which resolving does not deteriorate the performance of heuristic pricing policies. Maglaras and Meissner [11] show that resolving heuristics are also asymptotically optimal as starting inventory and expected sales both go to infinity and Cooper's example should not persist in problems with large demand potential. There is limited research on developing dynamic pricing heuristics and those that are suggested are usually based on deterministic formulations. The main contribution in this paper is to propose two new heuristics that are simple and computationally feasible. While dynamic run-out rate heuristic also uses the deterministic solution in feedback form, revenue approximation heuristic is based on approximating the revenue-to-go function using a homogeneity assumption.

The literature also does not provide enough guidance on non-asymptotic or average performance of heuristic policies and the factors that moderate their performance. In GvR, the authors use the exponential price sensitivity of demand and conduct a small numerical experiment to study the performance of the fixed-price policy against the optimal dynamic policy. It is shown that the revenue gap between the fixed-price and dynamic pricing policies is smaller than the theoretical bounds and gets smaller as starting inventory increases. However, Zhao and Zheng [12] show that the revenue gap is more significant when the constant demand elasticity function is used rather than the exponential demand function. Zhao and Zheng [12] also show that the revenue gap is rather insensitive to the elasticity of demand and there are diminishing marginal returns of dynamic pricing policies to the number of prices used. Maglaras and Meissner [11] conduct a numerical study on the multiproduct pricing problem with a linear demand function. Their results show that the fixed-price policy's regret over the optimal dynamic policy can be substantial and resolving the deterministic problem periodically during the horizon can offer significant benefits. In Section 3, we provide the results of an extensive numerical experiment to study the performance of heuristic pricing policies. The results show that the regret of fixed-price policies can be important in practice and dynamic pricing heuristics can be used to generate near-optimal results.

The remainder of this paper is organized as follows. In Section 2, we propose the revenue approximation and dynamic run-out rate heuristics. In Section 3, we report the results of a detailed numerical study that quantifies the regrets of fixed-price and dynamic pricing heuristics over the optimal dynamic pricing policy. This section also analyzes the effect of periodic price changes on the performance of dynamic pricing heuristics. We conclude in Section 4.

Table 1
The demand functions that are used in the analysis.

	$\lambda(p)$	$p(\lambda)$	$r(\lambda)$	λ^*	p^*
Exponential	ae^{-p}	$\ln\left(\frac{a}{\lambda}\right)$	$\lambda\ln\left(\frac{a}{\lambda}\right)$	$\frac{a}{e}$	1
Linear	$a-bp$	$\frac{a-\lambda}{b}$	$\frac{(a-\lambda)\lambda}{b}$	$\frac{a}{2}$	$\frac{a}{2b}$
Logit	$\frac{ae^{-bp}}{1+e^{-bp}}$	$\frac{1}{b}\ln\left(\frac{a}{\lambda}-1\right)$	$\frac{\lambda}{b}\ln\left(\frac{a}{\lambda}-1\right)$	$\frac{ae^{-W(1/e)-1}}{1+e^{-W(1/e)-1}}$	$\frac{W(1/e)+1}{b}$

2. Dynamic pricing heuristics

We first state our problem following the notation in GvR and provide some preliminary results. A given stock of n items is to be sold over a finite season of length t. The demand rate depends only on the current price p through a function $\lambda(p)$, whose inverse is $p(\lambda)$. The revenue rate, denoted by $r(\lambda)=\lambda p(\lambda)$, is assumed to satisfy $\lim_{\lambda\to 0} r(\lambda)=0$, and is continuous, bounded, concave and has a least maximizer denoted by $\lambda^*=\min\{\lambda : r(\lambda)=\max_{\lambda\geq 0} r(\lambda)\}$ (the corresponding price is $p^*=p(\lambda^*)$). There exists a *null* price denoted by p_∞ for which $\lim_{p\to p_\infty}\lambda(p)=0$. The price is selected from a set of allowable prices $\mathcal{P}=\mathbb{R}_+\cup p_\infty$. The corresponding set of allowable rates is denoted by $\Lambda=\{\lambda(p) : p\in\mathcal{P}\}$.

For the numerical examples and experiments in this paper, we use three different functions to model the price–demand relationship: exponential, linear and logit demand functions. These are some of the most commonly used demand functions in theory and practice [7,13] and are given in Table 1.[1]

The demand is stochastic and modeled as a Poisson Process. The firm controls the intensity at every instant by using a price in $\mathcal{P}$. The problem is to determine the pricing policy that maximizes the total expected revenue over the season denoted by $J^*(n,t)$.

For a given remaining time s and inventory x in the season, GvR show that the optimal expected revenue-to-go (and the corresponding optimal price at that instant) can be found by solving the following system of differential

$$\frac{\partial J^*(x,s)}{\partial s}=\sup_{\lambda}\{r(\lambda)-\lambda(J^*(x,s)-J^*(x-1,s))\},\quad \text{for all } x=1,2,\ldots,n, \tag{1}$$

with boundary conditions $J^*(x,0)=0$ for all $x=1,2,\ldots,n$ and $J^*(0,s)=0$ for all $s\leq t$. GvR also prove the existence of a unique solution to (1) along with monotonicity of the optimal expected revenue (and corresponding demand rates and prices) with respect to remaining inventory and remaining time in the season.

GvR state that obtaining a solution to (1) is quite difficult – if not impossible – for arbitrary demand functions. In addition, implementing a pricing policy that would change the price continuously over time may be difficult in practice. Therefore, they suggest the use of a heuristic pricing policy in which the price is constant for the entire season. The *fixed-price* (*FP*) heuristic that they develop uses the solution of the deterministic version of the problem and sets the price at $\overline{p}=p(\overline{\lambda})=p(\min\{\lambda^0,\lambda^*\})$, where $\lambda^0=n/t$ is the run-out rate and λ^* is the revenue maximizing rate. One can improve

[1] $W(.)$ denotes the principal branch of the Lambert W function, which is the inverse of the function $f(w)=we^w$. The numeric value of $W(1/e)$ is approximately 0.27846.