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Technical note

Data envelopment analysis: Prior to choosing a model

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ABSTRACT

In this paper, we address several issues related to the use of data envelopment analysis (DEA). These issues include model orientation, input and output selection/definition, the use of mixed and raw data, and the number of inputs and outputs to use versus the number of decision making units (DMUs). We believe that within the DEA community, researchers, practitioners, and reviewers may have concerns and, in many cases, incorrect views about these issues. Some of the concerns stem from what is perceived as being the purpose of the DEA exercise. While the DEA frontier can rightly be viewed as a production frontier, it must be remembered that ultimately DEA is a method for performance evaluation and benchmarking against best-practice. DEA can be viewed as a tool for multiple-criteria evaluation problems where DMUs are alternatives and each DMU is represented by its performance in multiple criteria which are coined/classified as DEA inputs and outputs. The purpose of this paper is to offer some clarification and direction on these matters.

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming based approach for measuring relative efficiency of decision making units (DMUs) that have multiple inputs and outputs [7]. Whether it is the researcher, the practitioner or the student, the use of the DEA methodology gives rise to some important questions before proceeding to a DEA analysis:

"What is the purpose of the performance measurement and analysis?"
"What are the decision-making units (DMUs) and the outputs and inputs to be used to characterize the performance of those DMUs?"
"What is an appropriate number of DMUs, given the number of inputs and outputs chosen?"
"What is the appropriate model orientation (input, output, additive)?"
"Does the analysis involve the use of ratio and raw data in the same model, and is this appropriate?"

We believe that within the DEA community, researchers, practitioners, and reviewers may have concerns and often incorrect views about these issues. The evidence for this concern materializes primarily in unpublished (and confidential) referee reports. It is, for example, very common practice, particularly in the case of novice users, to invoke the input-oriented constant returns to scale model in cases where inputs are in fact not under management control. More will be said regarding this below. Some of the concerns stem as well from what is perceived as being the purpose of the DEA exercise. While the DEA frontier can, in some situations, be rightly viewed as a production frontier, it must be remembered that ultimately DEA is intended as a method for performance evaluation and benchmarking against best-practice. The purpose of this paper is to offer some clarification and direction on some of these matters.

In the sections to follow we attempt to provide some guidance on, and possibly some answers to these questions.

2. Purpose of the performance measurement exercise

In any study of organizational efficiency it is necessary to have a clear understanding of the "process" being evaluated. A study of hospital efficiency, for example, must provide clarity as to which elements of the organization are being evaluated. Is it particular wards in the chosen hospitals (e.g. maternity wards), or particular functions such as emergency room procedures, or is it the cost effectiveness of the entire organization that is at issue? A clear specification of the function to be studied will drive the choice of inputs and outputs to be examined. A recent study of schools of business by Aviles-Sacoto [2], for example, placed considerable emphasis on the data gathering exercise aimed at understanding

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the precise measures deemed important by management. In that specific case it was international internships and job success, on the part of students, that were two of the most important factors for capturing school reputation. As discussed below, the "purpose" of the performance measurement exercise influences the model orientation. Numerous such examples are in abundance in the literature. (See, e.g., [3,9,19,27].)

3. DEA inputs and outputs

In the literature, DEA is generally introduced as a mathematical programming approach for measuring relative efficiencies of DMUs, when multiple inputs and multiple outputs are present. While the concept of inputs and outputs is well understood, it is often the case that researchers take the notion for granted, and little attention tends to be paid to insuring that the selected measures properly reflect, to the greatest extent possible, the "process" under study. While it is the case, as with regression analysis, that one can never be completely assured that all of the relevant variables have been included, every attempt should be made to include those that make practical sense for the setting under investigation. As a case in point, the original DEA model of Charnes, Cooper and Rhodes [7,8], involving the study of school districts in Texas, was developed in a ratio form of outputs/inputs, but the authors provide little in the way of rationalization in regard to appropriate variables (inputs and outputs) for studying student performance. This is not to imply that the variables used were not appropriate for the problem at hand, but rather it serves to illustrate that the paper, like many of those that followed over the past three decades, was primarily focused on methodological development. One gets the sense in much of the literature that there is little need to spend time laboring over how a process actually works. After all, in a production or service process, inputs and outputs are generally clearly defined. For example, the number of employees and profits are obvious examples of an input and an output, respectively.

Although DEA has a strong link to production theory in economics, the tool is also used for benchmarking in operations management, where a set of measures is selected to benchmark the performance of manufacturing and service operations. In the circumstance of benchmarking, the efficient DMUs, as defined by DEA, may not necessarily form a "production frontier", but rather lead to a "best-practice frontier". For example, if one benchmarks the performance of computers, it is natural to consider different features (screen size and resolution, memory size, process speed, hard disk size, and others). One would then have to classify these features into "inputs" and "outputs" in order to apply a proper DEA analysis. However, these features may not actually represent inputs and outputs at all, in the standard notion of production. In fact, if one examines the benchmarking literature, other terms, such as "indicators", "outcomes", and "metrics", are used. The issue now becomes one of how to classify these performance measures into inputs and outputs, for use in DEA.

In general, DEA minimizes "inputs" and maximizes "outputs"; in other words, smaller levels of the former and larger levels of the latter represent better performance or efficiency. This can then be a rule for classifying factors under these two headings. There are, however, exceptions to this; for example, pollutants from a production process are outputs, yet higher levels of these indicate worse performance. There are DEA models that deal with such *undesirable* outputs (see, e.g., [21,17].)

In certain circumstances, a factor can play a dual role of input and output simultaneously. For example, when evaluating the efficiencies of a set of universities, if one considers the numbers of Ph.D. students trained as outcomes from the education process, then this factor can rightly be viewed as an output. At the same time, however, Ph.D. students assist in carrying out research, and can therefore be viewed as a resource, hence an input to the process. See [12]. In such cases, the user must clearly define the purpose of benchmarking so that such performance measures can be classified as inputs or outputs. In some situations, the DMUs may have internal structures, e.g., a two-stage process. For example, banks generate deposits as an output in the first stage, and then the deposits are used as an input to generate profit in the second stage. In this case, "deposits" is treated as both output (from the first stage) and input (to the second stage).

In summary, if the underlying DEA problem represents a form of "production process", then "inputs" and "outputs" can often be more clearly identified. The resources used or required are usually the inputs and the outcomes are the outputs. If, however, the DEA problem is a general benchmarking problem, then the inputs are usually the "less-the-better" type of performance measures and the outputs are usually the "more-the-better" type of performance measures. The latter case is particularly relevant to the situations where DEA is employed as a MCDM (multiple criteria decision making) tool (see, e.g., [5,14,24]). DEA then can be viewed as a multiple-criteria evaluation methodology where DMUs are alternatives, and DEA inputs and outputs are two sets of performance criteria where one set (inputs) is to be minimized and the other (outputs) is to be maximized. In DEA, these multiple criteria are generally modeled as in a ratio form, e.g., the CCR ratio model [7]

$$\max e_{j_o}$$

$$\text{subject to } e_j < 1 \tag{1}$$

where

$$e_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$$

and x_{ij} and y_{rj} represents DEA *inputs and outputs*, and v_i and u_r are unknown weights. Obviously, x_{ij} and y_{rj} can be referred to in different terms, rather than "inputs" and "outputs". Inputs may, for example, be quality measures that act as surrogates for resources expended by the DMU. Outputs may, as well, appear in the form of outcomes such as employee satisfaction.

4. The numbers of inputs and outputs

It is well known that large numbers of inputs and outputs compared to the number of DMUs may diminish the discriminatory power of DEA. A suggested "rule of thumb" is that the number of DMUs be at least twice the number of inputs and outputs combined (see [16]). Banker et al. [4] on the other hand state that the number of DMUs should be at least three times the number of inputs and outputs combined. However, such a rule is neither imperative, nor does it have a statistical basis, but rather is often imposed for convenience. Otherwise, it is true that one loses discrimination power. It is not suggested, however, that such a rule is one that must be satisfied. There are situations where a significant number of DMUs are in fact efficient. In some cases the population size is small and does not permit one to add actual DMUs beyond a certain point. However, if the user wishes to reduce the number or proportion of efficient DMUs, various DEA models can help; for example, weight restrictions may be useful in such cases.

We point out that while in statistical regression analysis, sample size can be a critical issue, as it tries to estimate the average behavior of a set of DMUs, DEA when used as a benchmarking tool, focuses on individual DMU performance. In that sense, the size of the sample or the number of DMUs under evaluation may be immaterial.