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A game mechanism for single machine sequencing with zero risk

Mikhail Y. Kovalyov [a], Erwin Pesch [b,*]

[a] United Institute of Informatics Problems, National Academy of Sciences of Belarus, 220012 Minsk, Belarus
[b] Institute of Information Systems, Faculty III, University of Siegen, 57068 Siegen, Germany

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ABSTRACT

A problem is studied in which several non-cooperating clients compete for earlier execution of their jobs in a processing sequence of a single service provider in order to minimize job completion time costs. The clients can move their jobs earlier in a given sequence. They are assumed not to take a risky decision that can decrease their utility function. A game mechanism is suggested such that each client has no incentive to claim false cost and a social criterion is addressed, which is the minimum total cost of all clients. Algorithmic aspects of this mechanism are analyzed such as relations between the values of game equilibria and the social optimum, the computational complexity of finding a game equilibrium and the values of the price of anarchy and the price of stability.

1. Problem description

Many service providers such as logistics operators apply a *centralized decision* to determine the timing of their clients' processing. The clients are not always satisfied with the centralized decision and would like to have an influence on determining their positions in the processing sequence. This paper suggests and analyzes a *game decision mechanism* such that the clients are involved in the decision process, they are forced to be truthful if they do not accept a risky decision, and a *social criterion* is addressed.

We study a problem in which n clients, each having a single job, compete for earlier execution of their jobs in the processing sequence of a single service provider that processes jobs one after another with no idle time between them. Let $N=\{1,\dots,n\}$ denote the set of clients. We call job j the job of client j. The clients are assumed to be non-cooperative, that is, they cannot form coalitions to exchange information and generate a group decision. All jobs are ready for processing at time zero. A processing time p_j and a non-decreasing cost function $f_j(t)$ are associated with job j, $j=1,\dots,n$. Values p_j, $j=1,\dots,n$, are truly claimed by the service provider because his revenue from processing any job is fixed. The cost function $f_j(t)$ is claimed by client j and it can differ from his true cost function, $f_j^{true}(t)$, $j=1,\dots,n$.

Because of business confidentiality, values p_j, $j=1,\dots,n$, are not revealed to the clients, and the cost functions $f_j(t)$ and $f_j^{true}(t)$ of client j are not revealed to the other clients.

A game mechanism is suggested that determines an initial job processing sequence and rules of moving jobs to earlier positions in this sequence. If a job is moved, then the corresponding client is obliged to compensate the cost increase to the other clients whose jobs are shifted due to this move. After a certain number of moving operations, a final sequence is obtained and the jobs are processed in this sequence.

Given a final job sequence, let C_j denote the completion time of job j. It is equal to the sum of processing times of all jobs preceding and including job j. Client j aims at maximizing his *utility function* $F_j{:=}V_j^+ - V_j^- - f_j^{true}(C_j)$, where V_j^+ is the total compensation paid to him and V_j^- is the total compensation paid by him, $j=1,\dots,n$. Note that $\sum_{j=1}^n (V_j^+ - V_j^-)=0$.

We consider minimizing the total claimed cost of all clients, $\sum_{j=1}^n f_j(C_j)$, as the social criterion, which the service provider would like to address. The problem to find a job sequence that minimizes the total cost is denoted as $1\|\sum f_j(C_j)$ in the scheduling literature, see Graham et al. [22].

For an example, consider a cargo carrier company which possesses a sea liner to perform voyages from a single base port to several destination ports. Each voyage is direct in the sense that its route is from the base port to a destination port and back to the base port. At the beginning of a financial year, the carrier negotiates n long-term service contracts with the shipping companies. Each contract specifies delivery of a given cargo from the base port to one of the destination ports. The return trips are filled with cargo of the spot market. Due to the business constraints, cargo of different long-term contracts cannot be assigned to the same voyage. For the shipping company, each long-term contract j is associated with the cargo target delivery date D_j and an extra profit w_j, which the shipping company earns if and only if this cargo is delivered by D_j. For the carrier, a long-term contract is

* Corresponding author. Tel.: +49 271 7402420.
E-mail addresses: kovalyov_my@newman.bas-net.by (M.Y. Kovalyov), erwin.pesch@uni-siegen.de (E. Pesch).

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associated with the direct trip duration a_j, the backward trip duration b_j, and a fixed revenue. Revenues from the spot market contracts are not considered here. During the negotiation process, the carrier would like to work out a timetable for the n voyages which is satisfactory to the interested shipping companies, and then to specify it in the corresponding contracts.

In terms of the problem we study, the long-term contracts are the jobs, the total trip durations a_j+b_j are the job processing times p_j, and $f_j^{true}(t)=w_jU_j$ is the true cost function associated with the long-term contract j, where $U_j=0$ if $t-b_j \le D_j$, or equivalently, $t \le d_j:=D_j+b_j$, and $U_j=1$ if $t>d_j$, $j=1,\ldots,n$. Here, if job j completes at time t, then the corresponding cargo arrives to its port at time $t-b_j$. The problem of minimizing the sum of functions w_jU_j is denoted as $1\|\sum w_jU_j$, values d_j are called *due dates* and values w_j are called *weights* in the scheduling literature, see our definitions in Section 5.

Another example is the problem of determining an execution sequence for n computer tasks accepted by a computing service provider between two adjacent time points of making a decision of task acceptance or rejection and consequent execution of accepted tasks. Each accepted task j is associated with its execution time p_j and the money value w_j lost by the task owner per unit of time elapsed since the acceptance decision is made (time zero) and before its execution completes (time C_j). The owner of task j would like to minimize the total loss w_jC_j. The corresponding scheduling problem to minimize $\sum w_jC_j$ is denoted as $1\|\sum w_jC_j$.

Lately, there is a growing interest in decentralized supply chain management decisions and their comparison with centralized decisions, see, for example, Kaya [29], Hosoda and Disney [26], Jonrinaldi and Zhang [28], Varmaz et al. [49], Qiang et al. [43] and Zhang et al. [53]. Studies of decentralized decisions in scheduling dealt with such application areas as computer grids and clouds (Huang et al. [27]), parallel and distributed computer systems (Tchiboukdjian et al. [48]), crane scheduling (Sharif and Huynh [46]), semiconductor manufacturing (Yao et al. [52]), resource constrained multi-project scheduling (Homberger [25]), mobile robots scheduling (Giordani et al. [20]), and supply chain coordination (Qi et al. [42]). Game-theoretic models are often employed to handle decentralized scheduling problems. Relevant references are given in Section 4.

2. Game decision mechanism

We propose the following game mechanism for the service provider to find a job sequence which is satisfactory to all clients.

At the beginning, the clients claim cost functions $f_j(t)$, $j=1,\ldots,n$. The functions are assumed to be represented such that a constant number of elementary arithmetic operations is needed to calculate any value $f_j(t)$ for $j \in \{1,\ldots,n\}$ and $0 \le t \le \sum_{i=1}^{n} p_i$.

The mechanism is a decision process that generates a final job sequence. Firstly, an initial job sequence is generated. Any approach can be used here. For example, if the clients would like to have equal chances to take any position in the initial sequence, it can randomly be generated. If the clients agree that the mechanism applies any rule to generate the initial sequence, then we suggest that it is the best sequence with respect to minimizing the total claimed cost $\sum_{j=1}^{n} f_j(C_j)$, which the mechanism can find within a given time limit. The rules of developing this sequence can be known to the clients or not.

Then, if the initial sequence was developed aimed at minimizing the total cost, the mechanism updates the initial sequence by swapping the jobs in the first and the second positions. It is done to prevent any client from claiming a false cost function in order to take the first position such that it cannot be taken by any other job because of a high compensation payment.

Denote the updated sequence as $S^{old}=(i_1,\ldots,i_n)$. It is the input sequence for the first iteration of the decision process. Each client i_j, whose job is not in the first position, receives the completion time of his job in this sequence and a set of possible completion times obtained by placing his job in every earlier position assuming that the relative sequence of the other jobs remains unchanged. He also receives a set $E(i_j)$ of *eligible local strategies*. For a client i_j, whose job is not in the last position, this set includes every strategy (i_j,s) of moving his job to an earlier position s, $1 \le s \le j-1$, in S^{old}, such that his *claimed savings* emerged from applying this strategy are positive. The definition of claimed savings is given below.

Denote by S^{new} the sequence obtained from S^{old} by applying a job moving strategy (i_j,s), $1 \le s \le j-1$. The claimed savings of client i_j associated with this strategy are calculated as

$$D(i_j,s)=A-B, \quad A=f^{old}(i_j)-f^{new}(i_j), \quad B=\sum_{k=s}^{j-1}(f^{new}(i_k)-f^{old}(i_k)), \tag{1}$$

where value A is the reduction of the claimed cost of client i_j, value B is the *compensation* of client i_j to the other clients, and $f^{old}(i_k)$ and $f^{new}(i_k)$, $k=s,s+1,\ldots,j$, are the claimed costs of client i_k, calculated using completion time of job i_k in the sequences S^{old} and S^{new}, respectively.

If every job moving strategy brings non-positive claimed savings to client i_j, then only the "no move" strategy is eligible for him: $E(i_j)=\{(i_j,j)\}$. For the client, whose job is last, set $E(i_n)$ consists of the job moving strategies with positive claimed savings and the "no move" strategy (i_n,n) because staying last does not affect actions of other clients.

Each client i_j submits one eligible local strategy of the set $E(i_j)$, $2 \le j \le n$. The service provider selects and applies one of them. Again, any approach to the selection can be used here, for example, random selection. If the clients agree that the mechanism applies any selection rule, then we suggest that it selects the strategy that minimizes the total claimed cost $\sum_{j=1}^{n} f_j(C_j)$. The resulting sequence serves as the input sequence S^{old} in the next iteration of the decision process. When making a choice, the client can rank his eligible local strategies. For example, he may consider maximizing savings as the choice criterion. Alternatively, moving closer to the beginning of the sequence can be the choice criterion. Note that these criteria can be contradictory.

The decision process is repeated until a final job sequence is obtained for which no set $E(i_j)$, $j=2,\ldots,n$, contains an eligible job moving strategy different from "no move", or a decision time limit is exceeded. The jobs are processed by the service provider in the order determined by the final job sequence. All compensation payments are realized.

We call the described process as a *sequence updating game with compensations and jobs competing for earlier positions*. In this game, clients are *players*. We define an *Equilibrium* (*EQ*) of this game as a job sequence such that no client can obtain positive claimed savings by applying his eligible job moving strategy. This game can be classified as a non-cooperative game. Equilibria in a non-cooperative game have been studied by Nash [38]. The basics of the game theory and its terminology can be found, for example, in Osborne and Rubinstein [40].

3. Truthfulness of clients

In the described game the clients can claim false cost functions if there is no risk that this action will decrease their utility. Let us show that no client has an incentive to do this.

Consider a client j who lies about his cost function. Recall that all the cost functions are claimed before the sequence updating