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Neural Networks 18 (2005) 1332–1340

Neural Networks

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New conditions for global exponential stability of cellular neural networks with delays

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Received 7 April 2003; accepted 25 November 2004

Abstract

In this paper, we study further a class of cellular neural networks model with delays. By employing the inequality $a\prod_{k=1}^{m} b_k^{q_k} \le \frac{1}{r}\sum_{k=1}^{m} q_k b_k^r + \frac{1}{r}a^r \big(a \ge 0, b_k \ge 0, q_k > 0 \text{ with } \sum_{k=1}^{m} q_k = r-1, \text{ and } r > 1\big)$, constructing a new Lyapunov functional, and applying the Homeomorphism theory, we derive some new conditions ensuring the existence, uniqueness of the equilibrium point and its global exponential stability for cellular neural networks. These conditions are independent of delays and posses infinitely adjustable real parameters, which are of highly important significance in the designs and applications of networks. In addition, we extend or improve the previously known results.

Keywords: Cellular neural networks; Global exponential stability; Homeomorphism theory; Lyapunov functional; Equilibrium point

1. Introduction

Cellular neural networks (CNNs) was first introduced by Chua and Yang in 1988a,b. A cellular neural network is a nonlinear dynamic circuit consisting of many processing units called cells arranged in a two-or three-dimensional array. The structure of a cellular neural network is similar to that of a cellular automata in which each cell is connected only to its neighboring cells. A cell contains linear and nonlinear circuit elements, which typically are linear capacitors, linear resistors, linear and nonlinear controlled sources, and independent sources. The dynamical behaviors of CNNs and CNNs with delays (DCNNs) have received much attention due to their potential applications in associative memory, parallel computation, pattern recognition, signal processing, and optimization problems (see for example, Huang, 1996, 2001, 2004; Huang, Horace & Chi, 2004). When a neural circuit is employed as an associative memory, the existence of many equilibrium points is a necessary feature. However, in applications to solve optimization problems, the networks must posses a unique and globally asymptotically stable (GAS) equilibrium point for every input vector. Thus, a rigorous qualitative analysis of the GAS equilibrium point for CNNs and DCNNs is of great interest, and has been the concern of many authors (see, for example, Cao, 1999; Cao, 2003; Cao & Zhou, 1998; Li, Huang & Zhu, 2003; Lu, 2000). On the other hand, in the analysis of dynamical neural networks for parallel computation and optimization, to increase the rate of convergence to the equilibrium point of the networks and reduce the neural computing time, it is necessary to ensure a desired exponential convergence rate of the networks trajectories, starting from arbitrary initial states to the equilibrium point which corresponds to the optimal solution. From the viewpoint of mathematics and engineering, it is required cellular neural networks have a unique equilibrium point which is globally exponentially stable (GES). Thus, the analysis of the existence, uniqueness of the equilibrium point and its global exponential stability is of is of great importance for both practical and theoretical purpose. However, to the best of the author's knowledge, few authors have considered the existence, uniqueness of the equilibrium point and its global exponential stability for DCNNs (Cao, 2000; Cao & Wang, 2005; Huang, Cao & Wang, 2002; Mohamad & Gopalsamy, 2003).

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0893-6080/$ - see front matter
doi:10.1016/j.neunet.2004.11.010

Based on the above discussion, in this paper, we investigate further DCNNs model which can be described by delayed differential equations (namely, functional differential equations), and give a family of new sufficient conditions ensuring the existence, uniqueness of the equilibrium point and its global exponential stability by employing the inequality $a\prod_{k=1}^{m} b_k^{q_k} \le \frac{1}{r}\sum_{k=1}^{m} q_k b_k^r + \frac{1}{r}a^r (a\ge 0, b_k \ge 0, q_k > 0)$ with $\sum_{k=1}^{m} q_k = r-1$, and $r>1$ (Mitrinovic & Vasic, 1970), constructing a new Lyapunov functional, and applying the Homeomorphism theory (Forti & Tesi, 1995). These conditions do not require the signal functions to be differentiable, bounded and monotone nondecreasing. Moreover, the conditions obtained are also very easily checked in practice, and possess an important leading significance in solving optimization problems and reducing the neural computing time. We extend or improve the previously known results.

2. DCNNs model and preliminaries

Throughout the paper, R^n and $C[X,Y]$ denote the n-dimensional Euclidean space and a continuous mapping set from the topological space X to the topological space Y, respectively. Especially, $C \triangleq C[[-\tau,0],R^n]$.

Consider the model of DCNNs described by the following functional differential equations

$$\begin{cases} \dot{u}_i(t) = -c_i u_i(t) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t)) \\ \qquad + \sum_{j=1}^{n} b_{ij} g_j(u_j(t-\tau_{ij})) + I_i, \quad t \ge 0 \\ u_i(t) = \phi_i(t), \quad -\tau \le t \le 0, \end{cases} \tag{1}$$

where $i=1,\dots,n$; $c_i>0$; τ_{ij} are delays with $0\le\tau_{ij}\le\tau$; ϕ_i are assumed to be bounded and continuous functions on $[-\tau, 0]$; a_{ij} and b_{ij} are the synaptic connection strengths; f_j and g_j represent the neuronal output signal functions; I_i are the exogenous inputs.

Suppose that system (1) has a continuous solution denoted by $u(t, 0, \phi)$ or simply $u(t)$ if no confusion should occur, where $u(t)=\mathrm{col}\{u_i(t)\}$. For $u\in R^n$, we define the vector norm $\|\cdot\|$ and $\|\cdot\|_\infty$, respectively, by $\|u\| = \left(\sum_{i=1}^{n} |u_i|^2\right)^{1/2}$, $\|u\|_\infty = \max_i\{|u_i|\}$. For any $\phi=\mathrm{col}\{\phi_i\}\in C$, we define a norm in C by $\|\phi\|_\tau = \sup_{-\tau\le\theta\le 0}\|\phi(\theta)\|_\infty$. The signum function $\mathrm{sgn}(\rho)$ $(\rho\in R)$ be defined as 1 if $\rho>0$; 0 if $\rho=0$; -1 if $\rho<0$.

Definition 1. The equilibrium point $u^* = \mathrm{col}\{u_i^*\}$ of system (1) is said to be GES, if there are constants $\lambda>0$ and $M\ge 1$ such that

$$\|u(t) - u^*\|_\infty \le M\|\phi - u^*\|_\tau e^{-\lambda t},$$

for $\forall t \ge 0$.

Definition 2. If $f(t)$: $R\to R$ is a continuous function, then $D^+f(t)/\mathrm{d}t$ is defined as

$$\frac{D^+f(t)}{\mathrm{d}t} = \overline{\lim_{l\to o^+}}\frac{1}{l}(f(t+l) - f(t)).$$

Definition 3. (Forti & Tesi, 1995). A map H: $R^n\to R^n$ is a homeomorphism of R^n onto itself if H is continuous and one-to-one and its inverse map H^{-1} is also continuous.

Lemma 1. (Forti & Tesi, 1995). Let H: $R^n\to R^n$ be continuous. If H satisfies the following conditions

(1) $H(u)$ is injective on R^n.
(2) $\|H(u)\|\to\infty$ as $\|u\|\to\infty$.

Then H is a homeomorphism.

Lemma 2. (Mitrinovic & Vasic, 1970). For $a\ge 0$, $b_k\ge 0$ $(k=1,\dots,m)$, the following inequality holds

$$a\prod_{k=1}^{m} b_k^{q_k} \le \frac{1}{r}\sum_{k=1}^{m} q_k b_k^r + \frac{1}{r}a^r,$$

where $q_k>0$ $(k=1,\dots,m)$ is some constant, $\sum_{k=1}^{m} q_k = r-1$, and $r>1$.

In the paper, we introduce the following assumptions.

(H_1). There are constants $\mu_j>0$ and $\sigma_j>0$ $(j=1,\dots,n)$ such that

$$|f_j(u) - f_j(v)| \le \mu_j|u-v|, \qquad |g_j(u) - g_j(v)| \le \sigma_j|u-v|,$$

for any $u,v\in R$, and $j=1,\dots,n$.

(H_2). There exist constants α_{kj}, $\beta_{kj}\in R$, $q_k>0$, and $d_i>0$, $i,j=1,\dots,n$; $k=1,\dots,m$, such that

$$rc_i > \sum_{j=1}^{n}\sum_{k=1}^{m} q_k|a_{ij}|^{(r\alpha_{kj}/q_k)}\mu_j + \sum_{j=1}^{n}\sum_{k=1}^{m} q_k|b_{ij}|^{(r\beta_{kj}/q_k)}\sigma_j + \frac{1}{d_i}\left[\sum_{j=1}^{n}|a_{ji}|^{r\alpha_{m+1,i}}d_j\mu_i + \sum_{j=1}^{n}|b_{ji}|^{(r\beta_{m+1,i})}d_j\sigma_i\right],$$

where

$$\sum_{k=1}^{m+1}\alpha_{kj} = 1, \qquad \sum_{k=1}^{m+1}\beta_{kj} = 1,$$

$$\sum_{k=1}^{m} q_k = r-1, \quad r\ge 1, \quad i,j = 1,\dots,n.$$

3. Existence and uniqueness of the equilibrium point

In order to study the existence and uniqueness of the equilibrium point, we consider the following algebraic equations associated with system (1)

$$-c_i u_i + \sum_{j=1}^{n} a_{ij} f_j(u_j) + \sum_{j=1}^{n} b_{ij} g_j(u_j) + I_i = 0, \quad i = 1,\dots,n. \tag{2}$$